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I. *The Ice Calorimeter. With Remarks on the Constancy of the Density of Ice. By* Ezer Griffiths, *B.Sc., Fellow of the University of Wales.*

Received July 17, 1913.

I. Introduction.

The primary object of the work recorded in this Paper was the redetermination by an electrical method of the constant of Bunsen's ice calorimeter—*i.e.*, the mass of mercury drawn into the instrument by the addition of one mean calorie of heat.

Apart from the intrinsic value of this constant in calorimetry, its redetermination by an independent method is desirable on account of the marked discrepancy between Dieterici's value of J, determined by the ice calorimeter, and that found by other observers using entirely different methods. Also, Bunsen's calculated value of the latent heat of fusion of ice is 0·3 per cent. higher than that obtained in direct determinations.

Table I. is, I believe, a complete summary of the values hitherto published for the calorimeter constant.

Table I.

Authority.	K in mgms.	Reference.
Bunsen..................	15·41	" Phil. Mag.," XLI., 1871.
Schuller and Wartha .	15·44	" Wied. Ann.," 2, 1877.
Than......................	15·42	" Chem. Ber.," 10, 1877.
Velten	15·47	" Wied. Ann.," 21, 1884.
Zakrzevski...............	15·57	" Bull." l'Acad. Cracovie, 1891.
Staub......................	15·26	Inaug. Diss. Zürich, 1890.
Dieterici..................	15·49	" Ann. Phys.," 16, 1905.

Reference to the original memoirs, where available, shows that the values are based on observations of the heat imparted to the calorimeter by a small quantity of water contained in an envelope whose thermal capacity is comparable with that of the contained water.

Moreover, in the majority of cases no attempt has been made to vary the conditions, and thus detect systematic errors. For example, Bunsen's value is the mean of two experiments repeated under precisely similar conditions; the glass envelope weighed 0·2 gm., contained 0·3 gm. water, and a platinum sinker (weight 0·5 gm.) was also attached.

Of the determinations quoted in the above table Dieterici's would appear to carry the greatest weight as the quantity of water was varied from 0·6 gm. to 2 gms. His value, however, differs from the mean of the others by 0·4 per cent. Dieterici has also performed experiments in which the heat was supplied electrically, with the object of determining J in terms of the mean calorie. Assuming J as 4·184,* we have from his first series,† the value 15·27 for the calorimeter constant, and from the second series ‡ the value 15·46. The second series, however, is the more reliable, as the electrical quantities (E and C) were determined with greater accuracy by potentiometer measurements.

On account of the slow rate of heat supply the correction for "drift" § was considerable, amounting to as much as 1 per cent. in some of his experiments; and, moreover, it appears to have been very erratic, being positive in some experiments and negative in others.

II. OUTLINE OF THE METHOD USED IN THIS INVESTIGATION.

Although all electrical methods in calorimetry are dependent on the accuracy of the determinations of the electrical standards and of J, in this particular application they are free from two sources of error inseparable from the direct determination just discussed; loss in transfer and correction for the thermal capacity of the envelope.

Moreover, if the value assumed for J is stated and the electrical units are expressed in terms of international standards the value of the calorimeter constant can at any time be corrected with ease and certainty.

In the experiments described herein the results are based on determinations of electromotive force and resistance; but neither of these quantities had to be measured during the actual course of an experiment.

The current was so adjusted that the difference of potential at the ends of the heating coil was exactly equal to the E.M.F. of a number of standard cadmium cells in series, the equality of the balance being indicated by the non-deflection of a sensitive galvanometer in the standard-cell circuit.

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* *See* p. 4, §VI.

† "Wied. Ann.," 33, 1888.

‡ "Ann. Phys.," 16, 1905.

§ The term "*drift*" is used to denote the progressive freezing or thawing of the ice mantle independently of the measured energy supply.

All adjustments were made on a duplicate coil some time before the commencement of an experiment ; the change-over being effected by a mercury key which also recorded on a chronograph tape.

During an experiment the equality of the potential balance could be maintained with great exactness by means of a high resistance rheostat forming a shunt on an adjustable low resistance in the main circuit.

The conditions were varied as much as possible, E.M.F. due to 3, 4, 5, 6, 7 and 8 standard cells being balanced.

Thus the rate of energy supply in the fastest experiments was more than seven times that in the slowest.

Considerable precautions were taken to reduce the magnitude of the " drift." In some experiments the correction was negligible, and on the average it only amounted to 1 part in 500.

In order to understand the arrangement of the apparatus it is necessary to consider in some detail the sources of error peculiar to ice calorimeters and the precautions necessary for accurate work with these instruments.

III. SOURCES OF ERROR.

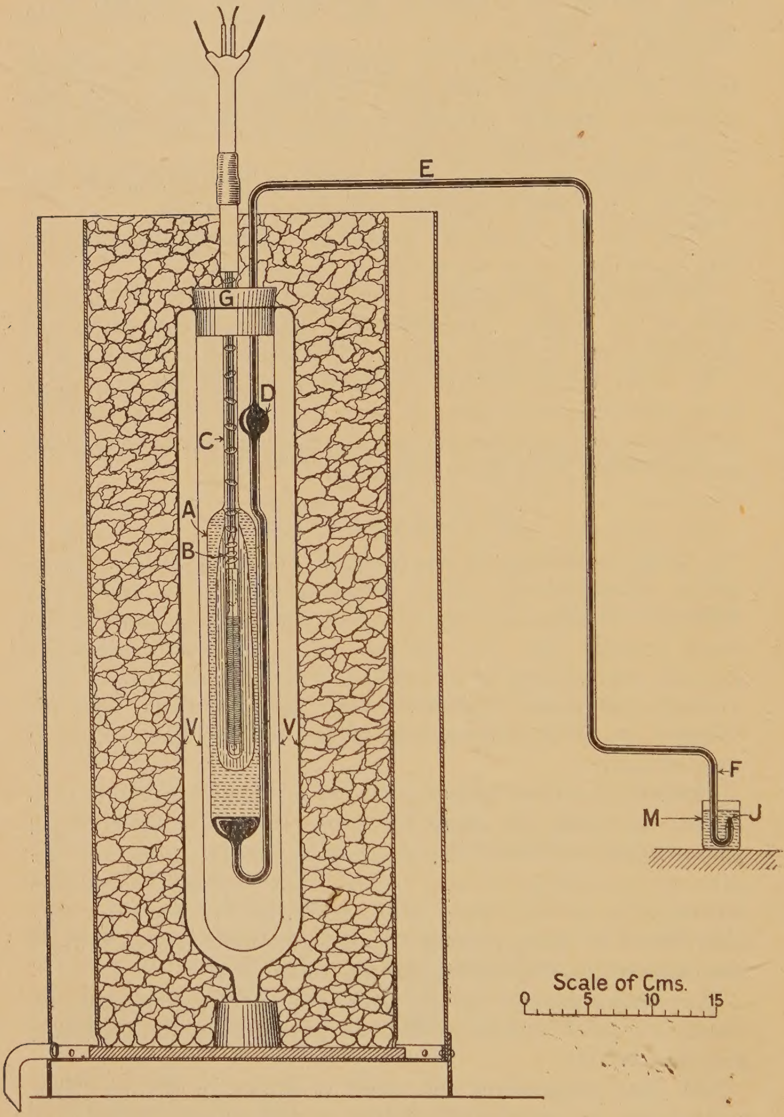
" *Drift.*"—Bunsen, in his classical Paper on the ice calorimeter, describes this effect, and attributes it to the slight difference—caused by impurities—in the melting point of the snow outside and the mantle inside the instrument. He devised the method of correction which has since been universally adopted. Unfortunately, however, the rate of " drift " is not a uniform one, so the correction is only an approximation.

Prof. Boys showed that the rate of drift could be reduced to about one-tenth its value by fixing the calorimeter in an empty vessel surrounded by ice, the effect being to reduce the rate of interchange of heat between the instrument and the surrounding ice or snow. Prof. Callendar further improved this device by using a second glass bulb fused around that of the calorimeter, and exhausting the outer bulb.

This arrangement is undoubtedly very satisfactory when the vacuum is good, but the difficulties of construction and filling are considerable.

The plan adopted in this work is shown in Fig. 1. The calorimeter is suspended within a transparent cylindrical vacuum vessel, the mouth of which is closed by a rubber cork fixed to the calorimeter.

The heat insulating property of the vacuum vessel, as also its transparency, were invaluable in this investigation, for it was desirable to watch and control the formation of the ice mantle. The cylindrical form of the vessel rendered its withdrawal



and replacement in the packing of powdered ice a simple matter provided the packing had been well rammed down.

Pressure Lowering the Melting Point.—In the customary form of ice calorimeter there is a difference in level between the surface of the mercury in the bulb and that in the capillary tube, sometimes amounting to 50 cm. In the first design of this calorimeter the capillary terminated at the level E in Fig. 1. After the instrument had been set up and the condition allowed to become steady, observations of the position of the mercury meniscus in the capillary were repeated at intervals for several hours. It was invariably found that a minute drift was taking place in the direction indicating a thawing of the ice mantle.

This effect was ascribed to the external head of mercury, causing a depression of the melting point of the mantle below that of the surrounding ice.

Since the pressure amounted to three-quarters of an atmosphere the melting point would be lowered by $\frac{1}{200}$°C.

This pressure effect was eliminated by extending the capillary and making the level approximately the same inside and out.

No drift was perceptible over long periods when experiments were not performed, and the slight drift which took place during, and after, an experiment was undoubtedly due to slow conduction down to the calorimeter of the heat generated in the leads.

Density of Ice said to be Variable.—This is one of the objections most frequently quoted against the use of the ice calorimeter in work of precision.

If well grounded it seriously limits the possibilities of the instrument, and the evidence upon which it is based may be summarised as follows :—

Nichols (" Phy. Rev.," VIII., 1899) reviews the work of previous investigators on the density of ice and describes his own experiments. He concludes that the density of ice mantles, determined by weighing in petroleum, is 0·91615 ± 0·00009. This result agrees with the mean value deduced from different methods by Plücker and Geissler, Kopp and Bunsen (for a similar variety of ice) to four places of decimals.

His experiments on the causes of the variations in density of artificial ice were not completed. The method was to freeze the ice mantle around the innner tube of a calorimeter by pouring in a mixture of CO_2 and ether. The unfrozen water was shaken out as completely as possible, and the adhering water frozen, the remaining space being then completely filled with mercury. The weight of the mercury, together with that of the ice, gave the data for the computation of the density of the ice mantle.

Although the results were consistent among themselves the absolute value was subsequently found to be erroneous on account of the deformation of the glass vessel under the weight of the contained mercury.

Nichols at first thought the discrepancy—amounting to 1 per cent.—between his value and Bunsen's was due to the much lower temperature at which the mantle was formed in his own experiments. He, therefore, made some determinations with mantles frozen by means of alcohol at -5 deg. to -10 deg. as refrigerant in the manner devised by Bunsen. The measurements appeared to indicate that the mantles formed by the use of alcohol at -5 deg. to -10 deg. were less dense than those formed by means of CO_2 and ether at -70 deg. by at least 1 part in 1,000, and further that one of the latter mantles decreased in density by nearly this amount after standing 24 hours in an ice bath.

The use of CO_2 and ether resulted in a very rapid formation of ice and the mantle, when a certain size was reached, invariably became filled with a network of fine cracks.

Vincent * later took up the subject, and also investigated the coefficient of cubical expansion of ice.

He prepared the ice by means of a freezing mixture, and appears to have obtained a different density for each sample prepared. Table II. summarises all his results :—

TABLE II.

Experiment.	Density ice at 0 deg.	Weight assigned to the experiment.
1	0·916335	3
2	0·915460	2
3	0·916180	2
4	0·915540 0·916060	1 2
Weighted mean	0·9160	

His value for the coefficient of expansion of the above samples are consistent, and no connection between variation

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* "Phys. Rev.," XV., 1902.

in density and expansion can be traced. Vincent's mean value for the density is 1 part in 5,000 less than the mean of the results of Plücker and Geissler, Bunsen and Nichols.

Hence, we may sum up the evidence as follows :—

1. Observers, using entirely different methods, agree within 2 parts in 10,000 in assigning the value 0·9161 to the density of artificial ice.

2. That no other physical property, such as melting point or coefficient of expansion, appears to vary with the density.

3. That it has not been possible to obtain two or more definite and reproducible values for the density of artificial ice.

It would appear, therefore, that an explanation of the small variations in the density is to be sought for in the presence of occluded water * in the samples of ice examined.

Calculation shows that the presence of 1½ per cent. of water would be more than sufficient to account for the discrepancies observed by both Nichols and Vincent.

And in this connection we have the very significant fact that the value of the latent heat of fusion of ice obtained by calculation from the constants of the ice calorimeter exceeds the value obtained by direct determinations using ice in bulk by about 0·7 per cent.

The latent heat of fusion is the one physical property which would be seriously influenced by the presence of comparatively small amounts of occluded water.

On consideration it will be seen that the presence of occluded water in the ice mantle of a Bunsen's ice calorimeter would have no influence on the results obtained with this instrument.

"*Stiction.*"—A method which has been considerably used on account of its convenience is that of observing the position of the end of the mercury column in a graduated capillary tube.

Since, however, the meniscus is a receding one the readings are liable to considerable error on account of the phenomenon usually termed "stiction." Also it is necessary to calibrate the tube and to provide means of adjusting the position of the column in this tube.

The method of directly weighing the mercury drawn in is free from these objections, but the form of the orifice requires consideration, since the ingress of air bubbles into the tube must be prevented.

* The term "occluded water" includes any amorphous modification binding the crystalline mass together.

The difficulty appears to be due to a film of air which is trapped between the mercury in the orifice and that in the measuring cup when the latter is inserted in position; this film breaks the column and forms an air bubble in the tube.

It was found that experiments so affected would be consistent among themselves, but the absolute value would be as much as 1 per cent. too low compared with experiments when free from air bubbles. After many trials the form of jet * shown at J in Fig. 1, was developed.

A short piece of 2 mm. bore tubing was fused on at F. The end of this tube had been ground off to a sharp even edge; the bore being left parallel and unrestricted.

When the mercury cup M was inserted beneath and raised into position this form of orifice showed no tendency to trap air; the meniscus maintained a well-defined convex form; and any slight errors in the weight due to variations in the form of the meniscus would mean out in a series of experiments.

IV. DESCRIPTION OF THE APPARATUS.

The bulb A of the calorimeter was 27 cm. long by 4 cm. diameter; the inner tube B was 20 cm. long by 9 mm. diameter. The stem C (26 cm. long) was enlarged at its upper end to closely fit the head of the heating coil, a short piece of rubber effecting the joint.

The capillary † DEF was in one continuous length with a small bulb blown at D.

The usual method of attaching the capillary to the calorimeter by means of a cork, was avoided owing to the possibility of creep and the trapping of air bubbles.

The bulb D was inserted to prevent air bubbles being drawn into the main bulb when the calorimeter was allowed to warm up without a mercury cup at the orifice. The rubber cork G supported the instrument within a vacuum vessel. This vessel V was 52 cm. long by 6 cm. internal diameter.

The assembled apparatus was placed in a double walled can containing finely powdered ice, the water from the melting ice being allowed to drain away freely at K.

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* The form of jet devised by Schuller and Wartha was tried, but was unsatisfactory on account of the tendency of the mercury to drop completely out of the little bulb forming the orifice with slight vibration, caused by changing the mercury cups. This may possibly have been due to the opening being too large.

† The bore of this tube was 2 mm. as far as the bulb D, and from D on to F 1 mm.

Heating Coil.—The resistance in which the heat was developed was of bare manganin wire, of about 20-ohm resistance, wound on a mica rack of X form. The diameter of the coil was 7 mm., and as the internal diameter of the central tube B was but 9 mm., the transference of heat to the ice mantle was rapid. The windings extended a distance of 11 cm. along the rack, and two straight pieces of manganin connected the coil to the junctions of current and potential leads.

Both potential and current leads were of manganin to eliminate the thermoelectric forces ; the former leads were of thin wire, while each of the latter consisted of two thick wires in parallel to diminish the heating effect of the current in the calorimeter stem. The resistance of the coil alone without the leads could be determined "in situ." A small correction had to be applied to the value so determined on account of the change in resistance by the heating effect of the current in the experiments.

This correction factor had been investigated in the case of a duplicate coil and a table drawn up.

The small temperature coefficient of manganin made the correction of small magnitude ; the factor in the heaviest rate of supply only amounted to 1·0003. The coil was immersed in paraffin oil up to the junctions with current and potential leads.

V. METHOD OF FILLING THE CALORIMETER AND OF FORMING THE MANTLE.

Filling.—The calorimeter was fixed in an inverted position so that the capillary tube was at the highest point ; a funnel containing freshly boiled distilled water being attached to the orifice end of the capillary.

By repeatedly heating and cooling both bulb and tube could be completely filled with water, and finally the water was boiled a few times in the calorimeter. The desired quantity of mercury was introduced by heating and cooling with the orifice under mercury.

Freezing the Ice Mantle.—The preliminary cooling to 0°C. was effected by immersing the vacuum vessel and its contents in the ice packing for some time. For freezing a quill tube containing ether (cooled in liquid air) was inserted into the oil contained in the central tube of the calorimeter, the operation being repeated a considerable number of times.

Unless some ice was present in the bulb (from a previous freezing) considerable undercooling would take place, and then superfusion would set in suddenly, the entire space between the inner tube and the bulb becoming filled with a loose mass of ice plates and needles mixed with water. This fibrous mass was allowed to thaw until only a few nuclei remained, and then the cooling was recommenced.

The formation now was a well-defined mantle of ice growing around the tube B, and by varying the depth of immersion of the cooled ether tube the region of growth of the mantle could be varied at will.

Lowering a small Geissler thermometer into the oil in B, immediately on the withdrawal of the cooling tube, indicated that the interior was but very little below 0°C. in temperature.

Very rapid formation of the ice mantle is not desirable, as, for example, when liquid air was poured directly into the tube B the mantle produced was filled with large cracks, and serious strains set up in the ice and glass.

On completion of the freezing the oil was removed by suction, the interior washed a few times with fresh oil, and then the requisite amount of paraffin run in.

Three or four hours had to elapse before the conditions were sufficiently settled to proceed with an experiment.

After switching off the heating current the mercury cup was allowed to remain under the orifice for periods of 3 to 10 minutes before being removed for weighing.

Observations of the change in weight of the second mercury cup supplied the data for the computation of the drift during an experiment.

It was found that the rate of drift depended to a certain extent on the time which had elapsed since the previous experiment, and it appeared that the heat generated in the leads would continue to pass into the calorimeter for a considerable time after the current supply was stopped. In consequence more than two experiments could rarely be performed in one day.

VI. THE ABSOLUTE VALUE OF THE QUANTITIES INVOLVED IN THE MEASUREMENTS.

Value of J.—The direct determinations of the calorimeter constant given in Table I. are expressed in terms of the mean calorie. Hence for conversion of the electrical measurements to the same unit we require the value of J expressed in ergs per mean calorie. Two independent determinations of J in terms of the mean calorie have been made.

1. The direct determination by Reynolds and Moorby * gave the value of J as $4{\cdot}183\times10^7$ ergs. The range in Reynolds' and Moorby's experiments, however, was not exactly 0° to 100°, but on the average 1·2° to 1·4° to 100°. Since the variation of the specific heat of water with temperature near 0° is considerable a small correction † has to be appplied to the above number.

After such correction J in terms of the mean calorie becomes $4{\cdot}1836\times10^7$ ergs.

2. The indirect determination of Callendar and Barnes, based on measurements of electromotive force and resistance, gave the value 4·185 (Barnes, 1909).

Hence we may assume the value $4{\cdot}184\times10^7$ for J with the probability that the error does not exceed 1 part in 4,000.

Time.—An electrically driven seconds pendulum of Invar steel recorded seconds on a chronograph tape. The error of the pendulum was determined by comparison with a rated chronometer.

The change-over key (controlling the heating current) was so designed that any time-lag between the marking of the tape and the actual switching on was compensated for during the operation of switching off. The tape could be read to a twentieth of a second, so the probable error in the timing should not exceed 1 part in 5,000.

Mass.—The set of weights had been calibrated and the absolute value of the 100 grms. determined at the National Physical Laboratory.

The change in weight could be determined to 0·1 mgm.

Electromotive Force.—The set of standard cells were frequently compared by means of a Thomson-Varley potentiometer with two National Physical Laboratory standards BC—1 and 2. The E.M.F. of these cells at 20°C. was certified as 1·01830 international volts.

Resistance.—The resistance of the coil had been determined in international ohms at the National Physical Laboratory about nine months previous to this work. Its value had since increased by 1 part in 4,000, the change being probably due to slight oxidation and rough usage in the preliminary experiments.

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* "Phil. Trans.," A., 1897.

† "Thermal Measurements," E. H. Griffiths, p. 107.

VII. EXPERIMENTAL RESULTS.

Column 1.—Number of standard cells balanced at the ends of the heating coil.
,, 2.—Change in weight of mercury cup corrected for drift.
,, 3.—Time in seconds of the electrical supply.
,, 4.—Calorimeter constant in mgms. reduced to vacuo.
,, 5.—Difference of individual experiments from mean.

TABLE III.

No. Cd. Cells.	Weight of Hg.	Time (sec.).	Constant.	Difference.
3	2·4605	1,431·60	15·467	+0·019
3	2·1670	1,265·50	15·525	−0·039
4	4·5200	1,481·40	15·504	−0·018
4	3·4485	1,132·05	15·479	+0·007
4	3·1520	1,034·75	15·479	+0·007
4	2·5380	832·65	15·489	−0·003
5	3·2041	673·40	15·475	+0·011
5	5·1630	1,084·75	15·480	+0·006
6	2·3328	339·95	15·499	−0·013
7	4·2530	456·70	15·456	+0·030
7	3·3853	362·60	15·493	−0·007
8	9·6725	793·50	15·488	−0·002
		Mean	15·486	

Hence, the value of the calorimeter constant is 15·486, *with a probable error of* 0·08 *per cent.*

VIII. CALCULATION OF THE LATENT HEAT OF FUSION OF ICE.

It is of considerable interest to compare the value of the latent heat determined directly with that obtained from the ice calorimeter, assuming the density of mercury, water and ice at 0°.

Calorimeter constant	15·486 mgms.
Density of mercury at 0°	13·5956 gms. per c.c.
,, water ,, ...	0·99988 ,,
,, ice ,, ...	0·9161* ,,

Hence, calculated latent heat of fusion of ice

=80·30 *mean calories.*

Bunsen's value for the latent heat is 80·02 (assuming his own value for the density of ice and the calorimeter constant).

The most recent direct determination by Mr. A. W. Smith,[†]

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[*] The uncertainty of 1 part in 5,000 in the density of ice would have an effect of less than 1 part in 500 in the calculated value of the latent heat of fusion.

[†] "Phys. Rev.," 1903.

using artificial ice prepared from pure water, gives the value of the latent heat as 79·77, assuming J=4·184 and E.M.F. of Clark cell at 15° as 1·433 volts.

The difference of 0·7 per cent. between Smith's direct determination by an electrical method and the value calculated above, is difficult to explain on any other grounds than that of the presence of occluded water in the samples used for the direct determination.

If such is the case it illustrates the difficulty of obtaining ice entirely free from water, as Smith prepared the ice with considerable care.

SUMMARY OF RESULTS.

1. The evidence, adduced by various observers, in favour of ice having variable density cannot be regarded as conclusive.

2. The constant of the ice calorimeter is 15·486 mgms mercury per mean calorie assuming J=4·184 and E.M.F. of standard cadmium cell at 20° as 1·01830. This value agrees with Dieterici's determination, using water in quartz bulbs, to within 1 part in 3,000. Hence the discrepancy between Dieterici's value of J and that of other observers is not due to errors in his determination of the calorimeter constant, but is either due to inaccuracy in the absolute value of his electrical standards or systematic error in his experiments.

3. The calculated value of the latent heat of fusion of ice exceeds that obtained by measurements with ice in bulk by more than the probable experimental errors.

In conclusion, my thanks are due to Principal Griffiths, at whose suggestion the work was undertaken, and to my brother, Edgar A. Griffiths, for his assistance.

Research Laboratory, University College, Cardiff,
July 7, 1913.

ABSTRACT.

The primary object of the work was the re-determination, by an electrical method, of the constant of Bunsen's ice-calorimeter. The heat was supplied by a manganin coil wound on a mica rack which fitted the interior tube of the calorimeter, and the results are based on determinations of E.M.F. and resistance. The current was adjusted so that the difference of potential at the ends of the heating coil was exactly equal to the E.M.F. of a number of standard cadmium cells in series.

The conditions were varied as much as possible. Thus the rate of energy supply in the fastest experiments was more than seven times that in the slowest.

Errors due to progressive freezing or thawing of the ice mantle were greatly diminished by suspending the calorimeter within a transparent vacuum vessel of cylindrical form, the stem and capillary of the calorimeter projecting through a rubber stopper, and the vacuum vessel being completely embedded in powdered ice.

The mean value of the calorimeter constant was found to be 15·486 milligrammes of mercury per mean calorie.

The Constancy of the Density of Ice.—Various observers have advanced evidence tending to show that the density of ice at 0°C. is not a definite constant. A consideration of their work leads to the conclusion that the small variations of density found for different samples might be simply due to the presence of occluded water or an amorphous modification cementing the ice crystals together.

The value (80·30) of the latent heat of fusion of ice, calculated from the ice calorimeter, supports this view, as it is higher by about 0·7 per cent. than the value obtained by direct determinations with ice in bulk.

DISCUSSION.

Prof. H. L. Callendar noted with pleasure that the author had very closely verified Dieterici's value, using an entirely different method. He had had occasion to look into Dieterici's determinations in connection with the specific heat of water, and thought them very reliable. They agreed very closely indeed with the results obtained by Dr. Barnes and himself by the continuous electric method. The most uncertain element in Dieterici's work was the thermal capacity of his silica bulb, which amounted to about 25 per cent. of the whole. The extreme uncertainty of the calculation of latent heat from the volume changes on melting was not, perhaps, sufficiently realised. The calculation depended on the *difference* of the specific volumes of ice and water, so that any error in the value assumed for the density of ice seriously affected the value obtained for L. Thus the author, taking Vincent's result for the density of ice (=0·9161 gramme per cubic centimetre), had calculated L=80·3 *mean calories*, whereas, if 0·9167, the density as given by Barnes, had been assumed 79·5 (approx.) would have been obtained for the latent heat.

Dr. J. A. Harker mentioned a possible error due to the suction of mercury at atmospheric temperature, through a capillary of varying temperature, into the body of the calorimeter. Was the author's method of getting rid of "drift" superior to the method of putting one calorimeter inside another and larger one, and, by varying the pressure on the ice in the second one, adjusting its temperature until the "drift" was zero? He had at one time tried glacial acetic acid instead of ice. It had the advantage that when heat was supplied mercury was pushed out instead of sucked in. Also the transition temperature was between 16°C. and 17°C., and the constant was about 30.

Mr. F. E. Smith referred to the question of the leads to the coil. If they were too thick too much heat was conducted along them; if too thin too much heat was produced in them by the current. If the cross-section was assumed to be 2 sq. mm. then the resistance of a pair of manganin leads, each of which was 25 cm. long, would be 0·1 ohm. Since the heating coil was of 20 ohms resistance it followed that the heat produced in the leads was 0·005 times that produced in the coil. This heat was not accounted for in the calculations, and he would be glad if Mr. Griffiths would state what became of it. In his opinion much of it passed into the oil in the calorimeter. The experiments made by the author did not appear to him to indicate the magnitude of the effect, since the heat passing from the leads to the oil would be proportional to the time of an experiment and also to the square of the current used.

Prof. SILVANUS THOMPSON asked whether there was any evidence of an allotropic or amorphous form of ice. According to the theory of crystal formation by arrangements of densest packing of molecules, any amorphous form should be less dense (as is the case of fused silica as compared with quartz crystal) and its specific heat per unit mass should be greater. Was the author sure that the water used was perfectly air-free ?

Prof. C. H. LEES said the author had raised his opinion of the ice-calorimeter. He would like to know where it now ranked in the author's own estimation among other instruments used for similar purposes.

The AUTHOR agreed that the calculated value of the latent heat must be accepted with reserve, as it was difficult to predict in what direction the assumed value of the density of ice might be in error. The presence of occluded water in the specimens used would make the value too high, while minute air bubbles, due to dissolved air separating out, would make it too low. He had not yet attempted the substitution of other substances for ice. The use of glacial acetic acid would obviate the trouble at the orifice, since mercury would be ejected instead of sucked in. The heating coil was so designed that the heat generated in the leads was small. Moreover, only a fraction of this would pass into the bulb of the instrument, since the coil proper extended nearly the entire length of the inner tube. The heat generated in the leads outside the bulb would be partially absorbed by the glass stem and the air surrounding the leads. The slight conduction by the glass stem and leads down to the bulb would cause a minute " drift " lasting for a considerable time after switching off the heating current. This was corrected in the manner indicated in the Paper. He regarded the variation of the density of ice to be due to occluded water and not to allotropic crystallisation. He considered the water to be present as an amorphous cement binding the crystals together, In reply to Prof. Lees, he considered electrical methods, as, for example, the continuous electric method of Prof. Callendar, superior to the ice calorimeter. One advantage of the latter was, however, that there was no radiation correction.

II. *An Electrostatic Oscillograph. By* H. HO, *Prof. of Electrical Engineering, Imperial University, Tokio, and* S. KOTO, *Asst. Prof. of Electrical Engineering, Imperial University of Kyushu.*

COMMUNICATED BY MR. R. S. WHIPPLE.

RECEIVED SEPTEMBER 30, 1913.

Introductory.—As far as we know, the first attempt to record oscillations by an electrostatic force was made by E. T. Jones* in 1907. His instrument was connected in idiostatic fashion, however, and vibrated with a frequency twice as high as that of the impressed voltage, consequently it cannot be said to have much value as an oscillograph. In the discussion on E. A. Watson's Paper,† in 1910, J. T. Irwin showed some curves which were taken on an electrostatic oscillograph, but as no description either of the instrument itself or of the results obtained has been published since that time, we are not in a position to criticise his invention or compare it with our own. The present device, which was brought to a practical form two years ago in the laboratory of the Engineering College, Tokio Imperial University, is the outcome of an endeavour to supply the need for a suitable oscillograph for recording high voltages, which we felt keenly in the investigation of the electrical properties of insulating materials, and we give here a rough description of its general principles and construction in the hope of interesting some persons engaged upon the same line of research.‡

The present practice of recording very high voltages with an ordinary electromagnetic oscillograph with high non-inductive resistance connected in series with it, possesses many disadvantages, which are enumerated as follows :—

(*a*) The large volume and the high cost of the non-inductive resistance.

(*b*) The considerable loss of energy in the resistance, which makes the method totally unsuitable for cases in which the source of the energy supply is limited, as, for example, when using an influence machine, or when the disturbance due to the current taken by the oscillograph cannot be neglected.

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* " A Short Period Electrometer," " Phil. Mag.," 1907.

† " Losses of Transmission Lines due to Brush Discharge," " Journal " of I.E.E., 1910.

‡ The instrument is now constructed by the Cambridge Scientific Instrument Co., Ltd., England.

(*c*) The effect of the electrostatic capacity of the resistance, which is more and more perceptible as the size of the resistance is increased and the voltage becomes higher, the consequence being that the current flowing into it through the oscillograph may not be in phase with the voltage to be recorded.

We wish to draw particular attention to this last point. H. L. Curtis and F. W. Grover * have shown that the effective inductance of so-called non-inductive resistances on the market reached such a high value as —100,000 micro-henrys in a coil of 10,000 ohms. Needless to say, this would cause a considerable phase advancement of the current if such a resistance be used with an oscillograph, and the adoption of some special method of winding for minimising the capacity becomes imperative, which means very costly apparatus. In our electrostatic oscillograph the reduction of the voltage to be recorded to a value suitable for the vibrator is effected by means of condensers which are quite inexpensive and occupy a very small space, the power consumption being practically zero. The electrostatic capacity of the vibrator proper is exceedingly small, but oil-condensers in multiple with it are required to adjust its potential and a charging current is taken by them. It is, however, incomparably less than the current consumed by an ordinary oscillograph; hence, in most cases it causes no appreciable disturbance of the external potentials. The majority of the disadvantages can be thus eliminated by using an electrostatic oscillograph for high voltages, say, above 1,000 volts, there being no upper limit whatever, provided suitable condensers for series connection are used.

Description.—The general arrangement is shown diagrammatically in Fig. 1, in which the P.D. between a_0 b_0 is required to be recorded. C is an oil-condenser to be used, if necessary, for reducing the said voltage to a suitable value at the terminals a b of the oscillograph. Two bronze strips, s_1 and s_2, are stretched between two parallel metallic plates, F_1 and F_2, called "field plates"; these strips are parallel to one another and to the plates, and carry a small mirror m in the centre, the tension being adjustable in the manner usually adopted. Contrary to the method used in an ordinary oscillograph, the strips are insulated from each other by a silk thread, t, which connects them and passes over an ivory pulley, p, thus forming two electrically independent conductors in the electric field of the parallel—plate condenser F_1, F_2. The plate F_1 has an opening w_1, for the passage of the light ray to and from the mirror, and an exactly similar one is cut in the other plate, F_2, to maintain the symmetry of the electric field. The plates and the strips, mounted on an ebonite frame as shown in Fig. 2, complete the vibrator, which is immersed in an oil bath, provided with the necessary number of terminals with suitable insulating bushings for the pressure to be employed. Two oil-condensers, C_1 and C_2, in series are connected to the points $a\ b$, in multiple with the field plates, as in Fig. 1, one of which condensers must be adjustable.

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* "Bulletin" of the Bureau of Standards, Vol. VIII., No. 3.

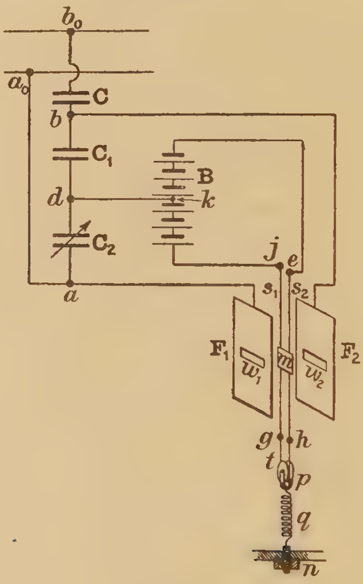

FIG. 1.

B is a water-battery, or a number of small dry cells, the latter being decidedly preferable, connected in series and mounted on an insulating stand, the electrical centre k of this battery being connected to the point d between C_1 and C_2. Its terminals are connected to the two strips s_1, s_2 in order to charge them for the same purpose as the needle of a quadrant electrometer is charged. The width of the field plates is 9 mm., the

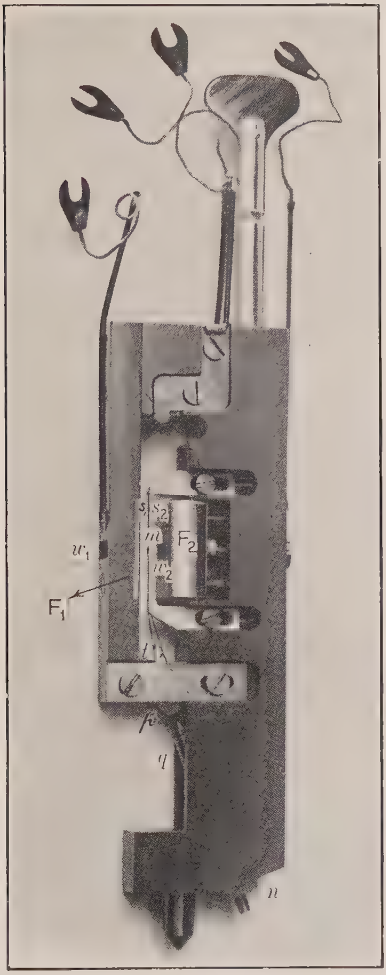

FIG. 2.

To face p. 18.

distance apart 5 mm., and the vibrator has approximately the same dimensions as the latest Duddell vibrator. These two types of vibrators can thus be placed conveniently side by side in the same oil bath, for which purpose one of the field plates F_2 is made of soft iron to serve as a part of the magnetic circuit.

Theory and Method of Use.—Fig. 3 represents two insulated strips, s_1, s_2, placed between two insulated parallel metallic plates, F_1, F_2. First, suppose equal and opposite charges $+q$ and $-q$ be now given to s_1 and s_2, a distribution of the charge is produced on the conductors F_1 and F_2—let us call it distribution No. 1—but their potentials remain at zero as before,

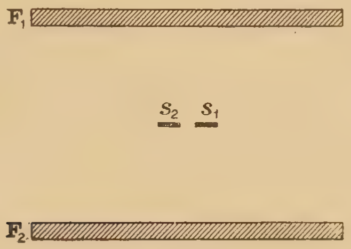

FIG. 3.

while the strips acquire equal and opposite potentials proportional to q, which may be denoted by

$$\left.\begin{aligned} v_{s_1}&=kq \\ v_{s_2}&=-kq \\ v_{f_1}=v_{f_2}&=0 \end{aligned}\right\} . \qquad \text{(1)}$$

Next, suppose no charge be given to the strips, but that a P.D. $(v'_{f_1}-v'_{f_2})$ be applied between the plates F_1 and F_2. The Faraday-tubes thereby produced do not differ appreciably from those obtained when no strips are present, as they are placed at right angles to the flow of the tubes and the thickness is very small compared with the distance between the plates. Since the strips are in the exact centre between the plates their potentials must then be

$$v'_{s_1}=v'_{s_2}=\tfrac{1}{2}(v'_{f_1}-v'_{f_2}). \qquad \text{(2)}$$

Let the distribution of electricity produced on F_1, F_2 in this case be called distribution No. 2; the total charge on each of the strips is, of course, zero.

Let the battery B on an insulated stand and the two condensers C_1 and C_2 now be connected up as in Fig. 4, the middle point of the E.M.F. of the battery being connected with the point *d* between the condensers, and suppose the P.D. $V_a - V_b$ be applied at *a b*, then evidently

$$V_{f_1} - V_{f_2} = V_a - V_b \qquad \text{. . . . . (3)}$$

$$V_{s_1} = \tfrac{1}{2}(V_a - V_b) + \tfrac{1}{2}E_B \qquad \text{. . . . . . (4)}$$

$$V_{s_2} = \tfrac{1}{2}(V_a - V_b) - \tfrac{1}{2}E_B, \qquad \text{. . . . . . (5)}$$

where E_B is the total E.M.F. of the battery. We wish to determine the distribution of electricity under this condition. In the case which we have first supposed, let the equal and opposite charges given to s_1, s_2 be in accordance with the equations

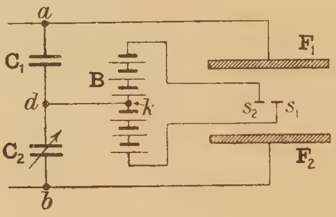

FIG. 4.

$$\left.\begin{aligned} q_{s_1} &= \frac{1}{k}\frac{E_B}{2} \\ q_{s_2} &= -\frac{1}{k}\frac{E_B}{2} \end{aligned}\right\} \qquad \text{. . . . . (6)}$$

Then, by (1)

$$\left.\begin{aligned} v_{s_1} &= \frac{E_B}{2} \\ v_{s_2} &= -\frac{E_B}{2} \end{aligned}\right\} \qquad \text{. . . . . (7)}$$

The distribution No. 1 is produced on F_1F_2, and their potentials (v_{f_1} and v_{f_2}) are both zero.

Next, in the case we have secondly supposed, let

$$v'_{f1} = V_a,$$

$$v'_{f2} = V_b,$$

then the distribution No. 2 is produced on F_1F_2, and the strips are at the potential

$$v'_{s1} = v'_{s2} = \tfrac{1}{2}\,(V_a - V_b).$$

Suppose these two distributions to be superimposed, the result must then produce another state of equilibrium, and the potentials must be as follows :—

$$(v_{f1}+v'_{f1})-(v_{f2}+v'_{f2})=V_a-V_b=V_{f1}-V_{f2},$$
$$v_{s_1}+v'_{s_1}=\tfrac{1}{2}(V_a-V_b)+\tfrac{1}{2}E_B=V_{s_1},$$
$$v_{s_2}+v'_{s_2}=\tfrac{1}{2}(V_a-V_b)-\tfrac{1}{2}E\ =V_{s_2}.$$

But, as may be seen, they are the same as (3), (4), (5), hence we may conclude that the electric distribution, corresponding to the condition expressed by (3), (4) and (5) is the superimposing of two such distributions as we have just described. That is to say, on the strips are superimposed :—

(α) Equal and opposite charges proportional to E_B.

(β) Induced charge which is zero in total, on each strip. On the plates are superimposed.

(γ) the distribution No. 1 corresponding to (α),

(δ) the distribution No. 2 proportional to (V_a-V_b).

Between (β) and (γ), between (β) and (δ) and between (α) and (γ) no force can be produced that gives rise to the deflection of the mirror, that between (α) and (β) being only influential in producing a turning moment, the momentary value of which is proportional to the product of the momentary values of E_B and (V_a-V_b). Hence, if E_B be constant, the deflection of the mirror will follow the variation of (V_a-V_b) accurately, provided proper damping is obtained. This gives a rough idea of the principle of the action of our electrostatic oscillograph.

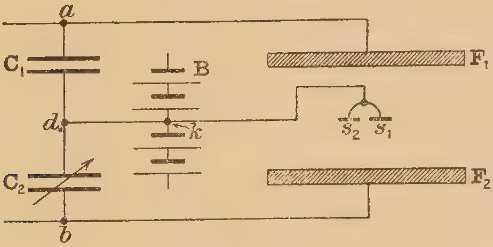

FIG. 5.

In the above we have supposed the pair of the strips to be in the geometrically exact centre between the field plates, but this is difficult to attain in practice. Consequently, it is found in practical use that it is not sufficient to make C_1 equal to C_2, but the following adjustment is always necessary. Disconnect the strips from the terminals of the battery, join the strips together and connect to the point *k*, as in Fig. 5, and apply an alternating E.M.F. at *ab*, which will cause the light spot on the screen to vibrate with a frequency twice as high as that of the E.M.F., as may be observed by the revolving mirror. This vibration is produced by the electrostatic force acting on the pair of strips, caused by a lack of symmetry and the small inevitable inequality of tension between the strips. Adjust one of the condensers, C_1C_2, till this vibration totally disappears ; this ensures that the potential of the strips will remain equal to that of the same position when there were no strips at all. The adjustment of the condenser being thus made, connect up as shown in Fig. 4, and the oscillograph will then be ready for use. If we impress an alternating E.M.F., the imperfect adjustment of the said condenser will manifest itself by the dissimilarity of the forms of the positive and negative waves, and the incorrect choice of the point *k* is discerned at once by the shifting of the wave from the zero line.

It is advisable to connect the metallic oil-bath to the point *d*, and when the vibrator is used with another electrostatic vibrator or with an electromagnetic vibrator in the same bath it must be shielded on both sides by tinfoil suitably insulated with mica plates, and kept at the potential of the point *d* in its own circuit. It is perhaps more convenient to use a separate oil-bath for each vibrator, the optical system being suitably modified.

Oil plays the most important part in our oscillograph ; not only does it serve as the damping agent and as the insulator, but its high specific inductive capacity as compared with air does much in increasing the sensitiveness, and but for its well-known property of causing very little phase difference between the electric intensity applied and the consequent displacement, the electrostatic oscillograph would have been an absolute impossibility.

In cases where the voltage is low but the source of energy is extremely limited, so that a sufficient current cannot be taken to actuate the ordinary oscillograph, we may employ the electrostatic vibrator and apply the voltage in question to the strips s_1s_2, while the terminals of a high-tension battery or of an influence machine, as in Fig. 8, are connected to F_1F_2 instead of to the strips.

Sensitiveness.—Those who are not familiar with the electrostatic phenomena and are apt to undervalue the force electrostatically produced, may have an incorrect idea of the sensitiveness of our oscillograph, and assume that it must have a very long natural period to get an appropriate deflection. To contradict this impression it will suffice to state that our vibrator, when adjusted for the natural period of $\frac{1}{3300}$ second, as determined experimentally by the resonance method and at the same time by calculation in accordance with the formulæ for the natural period of a bifilar oscillograph, gave a wave 2 cm. in amplitude at the distance of 70 cm. by $(V_a—V_b)$ =2,000 volts effective, the total E.M.F. of the battery being 300 volts. The dimensions of the field plates were 15 mm. by 9 mm., and the distance between them 5 mm.

The sensitiveness can be easily adjusted by changing the number of cells B. The proper range of the voltage at the terminals of the plates, when the connection of Fig. 1 is adopted, would appear to be 1,000—6,000 volts effective, although voltages up to 9,000 have been measured.

Parallel Working with an Ordinary Oscillograph.—Fig. 6 shows the curves obtained with a falling plate camera by using the electrostatic vibrator in parallel with an ordinary electromagnetic vibrator for a voltage of 2,000 volts effective, 60 ~ ; the curve A was produced by the electrostatic and B by the electromagnetic vibrator. In series with the latter vibrator we connected a "non-inductive" resistance of the type supplied with a wattmeter, as we had no suitable resistance specially made for oscillograph work, and the result is somewhat startling, showing in a most pronounced manner the effect of the capacity of the resistance by the advance of phase thereby produced, thus confirming the statement of Curtis and Grover in the Paper already referred to, and incidentally emphasising the superiority of the electrostatic oscillograph for high-voltage work. We have reason to believe that the high resistances supplied by oscillograph makers at present will give results not much different from this.

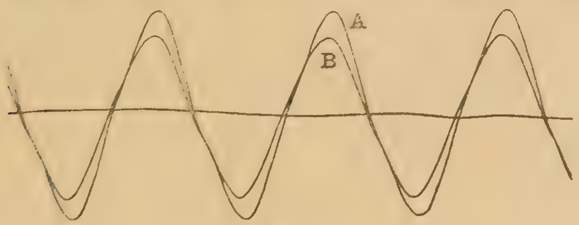

Fig. 6.

Fig. 7 shows the same experiment with a generator which gives a particularly pointed E.M.F. wave—the frequency being again 60 ∾.

Application to Recording Very Small Currents.—The recording of very small currents, lower than $\frac{1}{1000}$ ampere, say, is impossible with an ordinary oscillograph, unless a current transformer is used. But in some kinds of investigations the use of a current-transformer is not desirable, and the electrostatic oscillograph may be called upon again for this kind of service with advantage. The connection for this purpose is shown in Fig. 8, in which R_1 and R_2 are two exactly equal resistances with as little inductance and capacity as possible, connected in series in the circuit of the current to be recorded, the middle point d being connected to k the centre point of the E.M.F. of the high-tension battery B. It is less expensive to use an influence machine with two equal condensers joined in series, as shown in Fig. 9, in place of the battery.

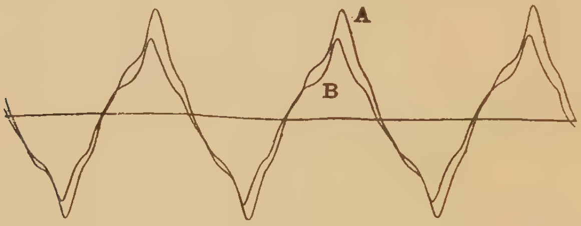

Fig. 7.

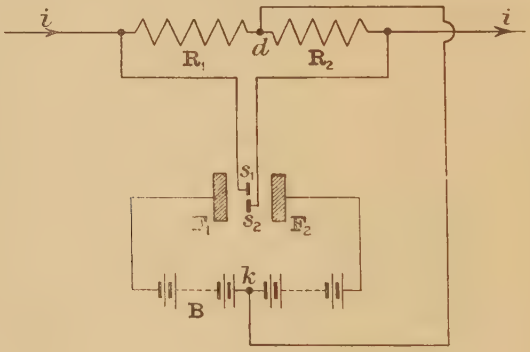

Fig. 8.

Fig. 10 is the result of an attempt to record the charging current while testing the dielectric strength of a glass plate 0·7 cm. thick between tinfoil electrodes 21 cm. by 15 cm. in air, with a pressure of 9,000 volts effective, the edges of the electrodes showing active brush discharges. A is the current curve taken in the said manner and B the voltage curve taken with another electrostatic vibrator. By passing a known current, the current curve was found to indicate a maximum height of about 0·0005 ampere ; the P.D. between F_1 and F_2 was kept at 3,500 volts with a Wimshurst machine. On account of the lack of a suitable number of condensers and resistances, the adjustment was not as satisfactory as could be desired, and the result is rather qualitative than quantitative, but it demonstrates the utility of the instrument for investigations in which small currents have to be recorded at very high voltages, and the loss of voltage in R_1 and R_2 can be neglected in comparison with the applied voltage.

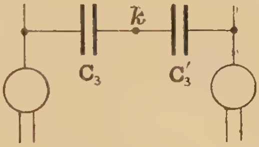

Fig. 9.

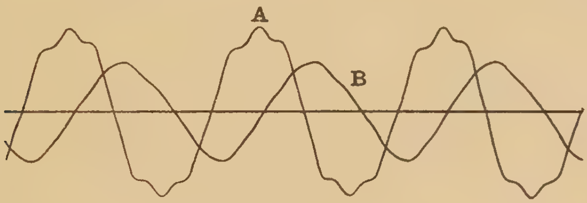

Fig. 10.

The Electrical Engineering Laboratory,
The Imperial University, Tokio, Japan.

ABSTRACT.

The Paper describes an electrostatic oscillograph suitable for recording very high voltages. Two vertical bronze strips pass symmetrically between two parallel metallic plates called "field plates." They are connected at their lower ends by a silk fibre which passes under an ivory pulley. An extremely small mirror is

fixed to the strips. This arrangement constitutes the vibrator, which, mounted on an ebonite frame, is immersed in an oil bath. To the upper extremities of the strips are connected the terminals of a direct-current voltage of about 300. The alternating voltage to be recorded is connected to the "field plates," in parallel with which there are two oil condensers in series. The electrical midpoint of the direct-current battery is connected to a point between the condensers.

The turning moment on the strips is proportional to the product of the momentary values of the alternating-current voltage and the direct-current voltage, so that if the latter is constant, the deflection of the mirror accurately follows the variation of the former.

Oil plays an important part, not only acting as a damping agent and insulator, but increasing the sensitiveness on account of its high dielectric constant.

In cases where voltage is low, but the source of energy is so limited that a sufficient current cannot be taken to actuate the ordinary oscillograph, the electrostatic vibrator may be used by applying the voltage in question to the strips, while the terminals of a high-tension battery, or influence machine, are connected to the "field plates." The instrument may also be used for recording very small currents by replacing the oil condensers by two exactly equal resistances, which are traversed by the current.

DISCUSSION.

Mr. A. CAMPBELL thought it was a considerable advantage to be able to do without high resistances. It had been suggested to him that the instrument might be of great use in work with X-rays and in other cases where the charge was being taken from an induction coil.

Mr. R. S. WHIPPLE emphasised the commercial advantage of the instrument. High resistances were very expensive. Einthoven was the first to propose an electrostatic oscillograph, and a Belgian inventor had constructed an instrument similar in principle.

Prof. G. W. O. HOWE stated that in the two curves given the difference was assumed to be due to a "lead" in the electromagnetic instrument. Was there any actual evidence that it was not due to "lag" in the other? If the oil did produce a phase-difference, it would introduce a "lag" and give the observed effect.

Mr. E. H. RAYNER (communicated): The oscillograph described by the authors and other similar instruments are intended to be instruments for depicting curves of current intensity. By the use of a series resistance the ordinary oscillograph gives a record of the voltage of a circuit. A watt-meter oscillograph would often give valuable information, as it is the power dissipated in a circuit which is often of primary importance, as in testing insulation, for instance. The authors' instrument appears to be usable as a watt oscillograph by substituting in Fig. 8 the secondary of a transformer for the polarising battery, the middle point of the winding being connected to the junction of R_1 and R_2 just as the battery is in the diagram. The primary of the transformer would be supplied from the alternator, which is connected to the circuit through R_1 R_2. There would only be a slight error due to the small difference from 180 deg. between primary and secondary of this transformer. If the alternator were a two-phase one the "idle" component of the current could be at once depicted by supplying the transformer from the other phase.

Mr. J. T. IRWIN said he was already aware of the device suggested by Mr. Rayner, having described it in 1907. The principal difficulty with an electrostatic oscillograph was that a very high polarising voltage is required if the zero is to be stable. The authors had reversed the process—using a small polarising voltage compared with the alternating-current voltage. Hence it is only when the alternating-current voltage is applied that a change of zero is produced. In most cases this cannot be compensated beforehand, as we do not know the value of the potential to be applied. Castor oil was not suitable for use in oscillographs, as it became acid and conducting under high voltages. Well-filtered paraffin had not this defect.

Mr. D. OWEN regretted that the Paper contained so little in the way of actual work accomplished with the instrument. If one took the values stated in the Paper for the inductance of the high resistances used with E.M. oscillographs, and assumed a frequency of 100∿ per second, calculation seemed to show that the phase-differences introduced by the use of such resistances were much too small to account for the discrepancies in the experimental curves.

Mr. G. L. ADDENBROOKE said that if castor oil free from dust and moisture were used he did not think oscillograms with the instrument would be affected perceptibly by action in the oil at 40 periods or higher. With a dry oil much lower periodicities might be used. The influence of moisture is greater with mineral oils than with castor. The dielectric strength varies almost directly with the percentage of moisture present.

Dr. G. W. C. KAYE (communicated): One of the features that interests me in the Paper is the possibility of applying the very neat and ingenious apparatus of the authors to the measurement of the potential curve of the current from an induction coil. Such a curve exhibits rapid and marked discontinuities and abruptness, and the difficulty of satisfactorily recording the detail by ordinary oscillograph methods has proved to be almost impossible. This is more especially the case when the coil is working an X-ray bulb, which ordinarily passes a current of the same order as that through the oscillograph itself. The various peaklets of the potential wave on an X-ray tube doubtless have their analogies in the cathode ray velocities; and it would be interesting to correlate the shape of the potential loop with the magnetic spectrum of the cathode rays in a coil-driven discharge tube.

Prof. KOTO, in reply, said that the frequency of the voltage used for the tests was 60∿, the records being taken on a falling-plate camera. He saw no objection whatever to the use of the oscillograph as a wattmeter, as suggested by Mr. Rayner and carried out by Mr. Irwin. He stated that the oil used was that supplied by the Cambridge Scientific Instrument Co., and that from some rough measurements he had made he found it to have a specific inductive capacity of about 2. The deviation of the strips from the centre was a cause of the change of zero, as Mr. Irwin suggested, if a large polarising voltage be applied. To suppress this effect the authors made the distance between the two field plates, consequently the distance between the strip and the plates, sufficiently great to make this inevitable slight deviation from the centre a very small percentage of the distance between the strip and the plates, and consequently this deviation from symmetry would cause no appreciable "out of balance" of the forces due to the polarising voltage acting upon the strips and effecting this change of zero. Referring to the effect of the oil on the phase relationship of the curves, Prof. Koto stated that he had no doubt that the curves obtained on the electrostatic oscillograph and reproduced in the Paper were correct, but he intended to make a series of experiments on different oils to see whether any serious phase differences are caused by the oils having different dielectric hysteresis constants.

III. *On the Thermal Conductivity of Mercury by the Impressed Velocity Method.** *By* H. REDMAYNE NETTLETON, *B.Sc., Assistant Lecturer in Physics at Birkbeck College.*

RECEIVED JULY 17, 1913.

1. INTRODUCTION.

FOR some time the author has been interested in temperature gradients modified by the movement of the conductor itself, and has already, with the aid of ordinary thermometers, obtained the general form of temperature curve down a column of mercury heated at the top, maintained cold at the bottom, and moving uniformly upwards from the lower to the higher source. Assuming the isothermals in such a moving column to be horizontal, he has obtained an expression for the thermal conductivity of mercury†; but, owing to the nature of the assumption and the necessity to resort to graphic interpolation, no accuracy other than that of the right order was claimed for the value obtained.

The present communication describes an apparatus so designed that with a single thermo-junction—carried by a cathetometer capable of rotation—the temperature may be found at any desired point within a wide vertical vacuum-jacketed tube containing mercury. As a result of experiments with flows of different magnitudes the conclusions have been arrived at :—

1. That the isothermals in a vertical column of mercury moving upwards from cold to hot are remarkably horizontal even when the disturbance of temperature gradient by flow is very large.

2. That, owing to this fact and the great accuracy with which Newton's law holds in a vacuum-jacketed vessel for the range of temperature necessary, the impressed velocity method is very suitable for determining the thermal conductivity of mercury at the temperature of the enclosure; the method, in fact, possessing all the advantages of continuous flow calorimetry.

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* The expenses of this research—other than the cost of the mercury—were defrayed by a Government grant received through the Royal Society.

† "Proc." Phys. Soc, London, Vol. XXII., 1910. Also "Phil. Mag.," April, 1910.

2. Statement of Theory Employed.

The differential equation pertaining to the distribution of temperature down a moving conductor has already been discussed by the author, and it suffices here merely to state the solution in the form most suitable for the present research.

Consider a column of mercury heated by steam at the top and maintained cold by ice at the bottom, and let the mercury be flowing uniformly upwards, m grammes per second crossing a horizontal section. Assume the isothermals horizontal, and let all temperatures be measured from the temperature of the enclosure taken as zero, and all distances, reckoned upwards as positive, be measured from the isothermal within the tube, which is at the same temperature as the enclosure, and which for brevity is hereafter referred to as the zero isothermal.

Let K denote the thermal conductivity of mercury,
s=the specific heat of mercury,
A=the cross-section of the tube employed,
p=the perimeter of the tube,
E=the Newtonian coefficient of emissivity.

Then the temperature θ at any distance, x, from the zero isothermal is given by the equation

$$\theta=\mathrm{M}e^{\frac{ms}{2\mathrm{KA}}\cdot x}\sinh\lambda x, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $\lambda=\sqrt{\left(\frac{ms}{2\mathrm{KA}}\right)^2+\frac{\mathrm{E}p}{\mathrm{KA}}}$, and is thus constant for any one flow, and M is a constant depending on the temperatures and positions of the fixed sources.

Since $\sinh\lambda(-x)=-\sinh\lambda x$, it follows at once that if θ_1 and θ_2 be the respective temperatures at distances L/2 above and below the zero isothermal, then

$$\frac{\theta_1}{-\theta_2}=e^{\frac{ms\mathrm{L}}{2\mathrm{KA}}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

the ratio of the two temperatures being thus independent of the heat lost by radiation.

If the mercury is at rest equation (1) becomes

$$\theta=\mathrm{M}\sinh\mu x, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where $\mu=\sqrt{\frac{\mathrm{E}p}{\mathrm{KA}}}$, and can easily be evaluated—as is well known—from observations of temperature at three equidistant points.

Though equation (2) is generally the most convenient for calculating the value of K, there is another very simple method of treatment showing the essence of the method—especially its relation to continuous flow calorimetry—and bringing out most clearly what method of procedure would be necessary if the experimental tube could not be regarded as uniform in cross-section.

Let g_1 be the value of the temperature gradient at a place where θ_1 is the temperature and A_1 the cross-section, and let g_2, A_2, θ_2 similarly refer to a lower position. We have then in the steady state

$$KA_1g_1-KA_2g_2=ms(\theta_1-\theta_2)+R, \quad . \quad . \quad . \quad (4)$$

where R is the heat lost by radiation between θ_1 and θ_2.

Now, $R=\int_{\theta_2}^{\theta_1} Ep\theta \, . \, dx$, and can be made approximately zero other than by taking θ_1 and θ_2 near together—namely, by choosing θ_1 and θ_2—so that the heat lost between θ_1 and the zero isothermal equals that gained between the zero isothermal and θ_2, a selection which resolves itself into choosing two areas approximately equal. If, then, by this means R be made negligible and the cross-section is sufficiently uniform, we have

$$K=\frac{ms}{A}\left(\frac{\theta_1-\theta_2}{g_1-g_2}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The maximum error made possible by the conductivity of the glass vessel containing the mercury and of the carrier within holding the thermo elements is easily found by considering the horizontal isothermals to extend into the material of the glass. If a be the area of the cross-section of the glass and k its thermal conductivity, the term $ka(g_1-g_2)$ should strictly be added to the left-hand side of equation (4). It follows that the values of K obtained either from equations (5) or (2), both of which can be deduced from the fundamental differential equation of the method, are too high, owing to the effect of the glass, but the error is necessarily less than 100 ka/KA per cent.

3. DESCRIPTION OF THE APPARATUS.

The experimental glass tube T containing the mercury is about 4·5 cm. in diameter and 60 cm. long, the upper portion being within the annular heater A; the middle portion of the tube is vacuum-jacketed for some 28 cm., and the lower portion dips into the large reservoir R of mercury which constitutes the lower temperature source. The object of the high vacuum is to secure constant conditions of emissivity without resort to cotton wool, it being a great advantage to see through the vessel prior to filling it with mercury at the commencement of each experiment. The iron reservoir R was made from a bottle of mercury by cutting off the top and cleaning and enamelling the inside; it rests on a tripod in the ice vessel V, which stands on the strong bridge B. This bridge also supports the annular heater carrying the experimental tube, which latter thus dips like a barometer or siphon into the mercury in R.

The thermo-junction of iron and constantan was prepared with skill by Messrs. Cossor, Limited. The thin insulated wires contained within the vertical glass tube *t* of about 6 mm. diameter were fused through the glass at *a* (Fig. 1) with inappreciable external distortion, the wires from *a* to the junction *b* being close together and in the same horizontal plane. The tube *t* was held at *Cl* by the steel carrier, as seen in Fig. 2, being bent and brought up along the carrier side, as shown, the wires emerging from the tube at F*e*, C*n*. The length *ab* was such that when the tube *t* was resting against the inner wall of the experimental tube T the point *b* would just reach the centre of the latter.

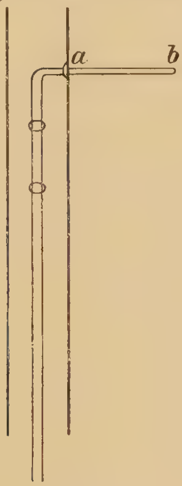

FIG. 1.

The cathetometer K, which carries the thermo-junction, was fitted with translational and rotational adjustments, and stood on the sector S, which itself could be rotated about a vertical axis, X, over the smooth glass track W laid out on the horizontal table L. Thus the straight portion *t* of the tube containing the thermo elements could be brought accurately into the vertical and into the same straight line as the axis of rotation.

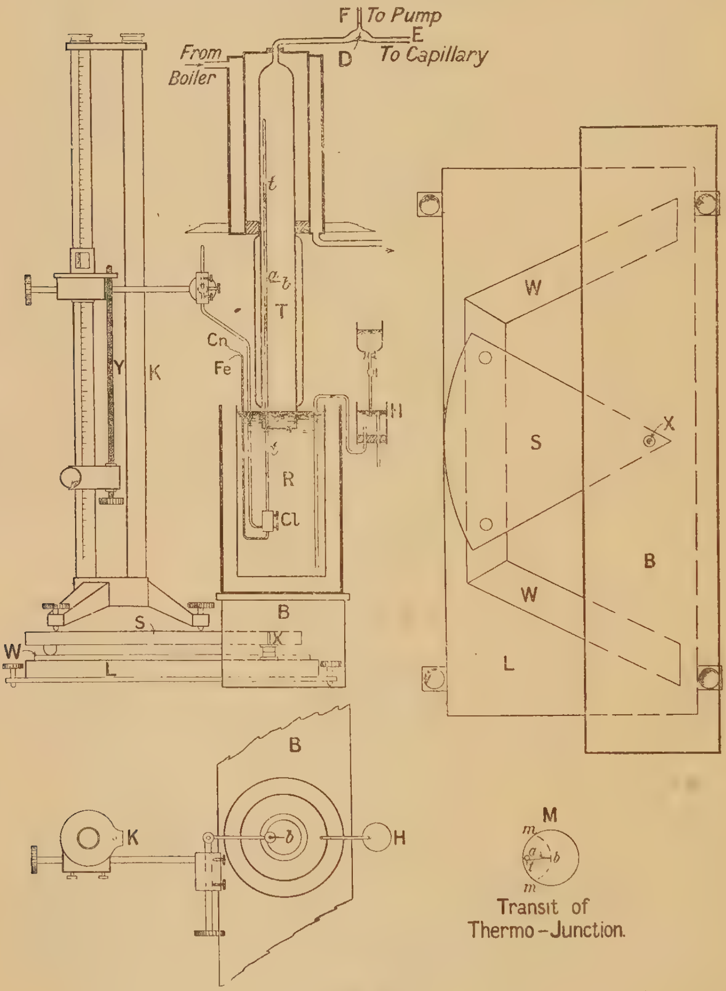

Fig. 2.

By turning the screw Y the tube *t* could then be raised or lowered vertically alongside the inner wall of the experimental tube T level, and situated as shown, while by rotating the sector S the tube *t* would rotate about its axis without any lateral displacement, the thermo-junction describing an arc, as seen in plan at M, the limiting position being when the actual junction touches the wall at *m*. In order that the longitudinal movement up or down of the tube *t* should in no way modify the distribution of temperature down the mercury column, it is necessary that, whatever be the position of *t*, its extremities should be well within the respective temperature sources A and R, a condition requiring that these constant sources and *t* itself should be of sufficient length.

The other junction of the thermo elements at the temperature of the enclosure was contained in a glass tube lying within a vessel of water in the proximity of the apparatus, while the joins of the iron wire to the galvanometer leads were likewise in glass tubes immersed side by side in water. The galvanometer was a quartz fibre instrument of the Broca type, and with the thermo-couple employed and the range of temperature to be measured deflections on the distant scale were accurately proportional to temperature changes.

The top of the experimental tube was narrowed down to capillary bore and attached by pressure tubing to the bulb tube D. At E was attached pressure tubing leading to a drawn-out capillary with its extremity well below the constant head H, this capillary constituting the resistance whereby a constant flow of mercury could be maintained up the tube T. Pressure tubing attached at F led to a Fleuss pump, enabling the whole experimental tube and capillary to be completely filled with mercury drawn up from the reservoir R. Convection currents around the annular heater were allayed by the free use of cotton wool. The Author would here record his thanks to Mr. J. M. Scholer, B.Sc., for his skill in adapting the cathetometer to permit the necessary movements and for the actual construction of the rotating table and its accessories.

4. Method of Experiment.

Before each experiment the table L was levelled to the horizontal, and the cathetometer and tube *t* to the vertical, *t* being also adjusted to be coaxial with X. The vacuum tube T was then let down over *t*, levelled and brought up so that *t* was just on the point of touching the walls. Mercury was then drawn up to the level of the thermo-junction, which, on rotating the sector S, was seen to sweep out the necessary arc without appreciable lateral displacement. The positions of the sector when the junction was touching the walls at *m* were carefully marked. All levelling adjustments being satisfactory, the apparatus was completely filled, and the experiment started, usually about eight o'clock at night. Steam was passed through A and ice placed in V, while a pinch cock was released and the height of the capillary adjusted to secure a suitable uniform flow of mercury. In two to four hours—according to the magnitude of the flow and the state of gradient when it was started—the steady state was attained, a shorter time being required than when the column was stationary.

The measurement of relative temperatures at definite places by the movement of the carrier tube *t* was now proceeded with and found to be satisfactory, for on moving to a definite cathetometer reading the junction registered its final temperature almost immediately. Moreover, on returning to the original position, the original temperature was recorded to an accuracy of the same order as the accuracy of re-setting. Starting with the thermo-junction in its highest position—the resistance of the circuit being adjusted to obtain a maximum deflection—it was lowered by carefully rotating the screw, readings being taken when the arrow head of the vernier lay on each whole and half centimetre division. The temperature curve was then retraced, *t* being raised from the lowest position to the starting point. The relative temperatures given in the results are obtained by adding the upward and downward readings. This process was tedious, and in the earlier experiments lasted an hour and a half; in later experiments many readings in the neighbourhood of the zero isothermal were omitted.

The flows were measured by the weight of mercury—ranging from 871 grammes to 1,422 grammes—outflowing in 15 minutes. They were weighed to the nearest gramme, and were at least constant to one part in five hundred. A flow of 1,000 grammes per hour is equivalent at 20°C. to a linear velocity of 17·84 cm. per hour. As, however, the density of mercury is not required, and varies slightly with the temperature, whereas the product of the density, the velocity and the area of the cross-section is everywhere constant, it is natural and convenient to measure the flow by weight.

The mean internal cross-section of the experimental tube T for the 12 cm. over which practically all measurements of temperature were made was found accurately after the research, with the tube t in situ, with the aid of distilled water and a cathetometer and needle. It was found to be 16·55 sq. cm., which agreed with the value found when t was removed and its cross-section found with a gauge and allowed for by subtraction. The experimental tube was found to be slightly conical, its cross-section varying by 1 per cent. in 11 cm. but as the extreme distances between readings used in calculating the thermal conductivity range from 6 cm. to 10 cm. the error is very small.

The effect of the upward flow is to make the actual range of temperature over the vacuum-jacketed portion of the tube T very much smaller than when the mercury is at rest. Calibration showed that for the 12 cm. explored the range varied from 20°C. to 25°C., according to the magnitude of the flow. In order, therefore, as a secondary experiment to find the value of Ep/KA from the temperature gradient when the mercury column is at rest, ether (B.P. 34·96 deg.) was circulated through the heater instead of steam. The temperature range over the investigated portion was then of the same order as in the dynamical experiments.

The specific heat of mercury is taken as 0·0333, which is Barnes'* value at 15°C.

5. Results of Experiments.

(*a*) *Horizontal Nature of Isothermals*.—On rotating the thermo-junction over a horizontal section there was no appreciable alteration of galvanometer deflection even when the junction was on the point of touching the walls and lying along the side of an inscribed hexagon. Many such rotations have been tried at different parts of the vacuum tube with flows of different magnitude, but all with precisely the same order of result; whereas on careful rotation of the junction the spot of light might move 1 mm. on the scale in either direction, on turning the screw to lower the junction vertically through 0·1 mm. a definite and larger effect would be produced. It is hence to be concluded that the isothermals in a vacuum-jacketed column of mercury are remarkably horizontal even when the temperature gradient is considerably modified by flow. It should here be pointed out that it is not perhaps surprising that even when the flow is swift no increased effect can be noticed, for the longitudinal result of a very swift flow over the explored portion of the tube is to render the range of temperature itself very small. The steep portion of the gradient under fast flows lies within a very small length within, or only just outside, the annular heater. The author is confident that the thermo-junction was in no way short-circuited, a second carrier tube and junction, registering temperatures somewhat lower down the vacuum tube, giving similar results.

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* Barnes, Brit. Assoc. "Report," p. 530, 1902.

(*b*) *Relative Temperatures with Flows of Varying Magnitude.* Table I. shows the chief results of the research—viz., the relative temperatures at definite positions of the cathetometer carrier.

(*c*) *Method of Calculating* K.—One typical example suffices to show the method of calculating K in each experiment. Usually five different values of $\log_{10}\frac{\theta_1}{-\theta_2}/\mathrm{L}$ are found, where L ranges from 6 cm. to 10 cm. It is clear that L should not be taken too small, for an error in observation of θ_1 or θ_2 would affect the ratio seriously.

TABLE II.—*Calculation of* K *from Readings of Experiment XI.*
Magnitude of Flow 1,099 *grammes per* 15 *minutes.*

L in cm.	Cathetometer readings	and deflections.	θ_1.	θ_2.	$\log_{10}\frac{\theta_1}{-\theta_2}/\mathrm{L}$.
10	76·0	41·15	48·60	−26·45	0·02642
	71·0	− 7·45			
	66·0	−33·90			
9	75·5	34·77	42·22	−24·36	0·02654
	71·0	− 7·45			
	66·5	−31·81			
8	75·0	28·79	36·24	−22·27	0·02643
	71·0	− 7·45			
	67·0	−29·72			
7	74·5	23·22	30·67	−19·99	0·02656
	71·0	− 7·45			
	67·5	−27·44			
6	74·0	17·98	25·43	−17·61	0·02660
	71·0	− 7·45			
	68·0	−25·06			
			Mean value	....	0·02651

From formula (2) of section 2 we now obtain directly K=0·0201.

It will be seen that θ_1 and θ_2 are reckoned from −7·45 as zero, the reading corresponding to the position 71·0 on the cathetometer. The true zero was at −6·0, a little—but never more than 2·5 mm. in the most extreme case—above the

TABLE I.—*Relative Temperatures at Definite Cathetometer Readin*

Number and date of experiment	I. Dec. 31 and Jan. 1.	II. Jan. 14 and 15, 1913.	III. Jan. 21 and 22.	IV. Jan. 21 and 22.	V. Jan. 28 and 29.	VI. Jan. 28 and 29.	VII. Feb. 4 and 5.	VII Feb. and
Magnitude of flow in grammes per 15 min.	1,296	1,159	1,199	1,003	1,422	1,079	No flow	1,36
Temperature of enclosure in deg. C.	14·0	15·0	15·0	15·0	15·5	15·8	13·1	15·
Reading corresponding to "zero temperature"	—7·0	—11·9	—5·7	—6·5	—10·0	—3·0	—15·0	—11
Cathetometer readings :—								
76·5	...	...	...	...	46·81	...	...	36·
76·0	48·35	...	47·44	...	38·30	...	...	28·
75·5	40·54	47·50	40·46	...	30·64	...	47·37	21·
75·0	33·43	40·45	34·01	46·85	23·87	47·84	...	15·
74·5	26·83	33·77	28·00	41·12	17·50	41·46	41·40	9·
74·0	20·83	27·50	22·24	35·60	...	35·42	...	...
73·5	15·10	21·60	16·87	30·58	6·60	29·63	35·68	...
73·0	9·80	16·23	12·00	25·90	...	24·27	...	...
72·5	4·95	11·21	...	21·60	— 3·20	19·27	30·08	—11·
72·0	0·31	...	2·98	17·00	— 7·46	14·80	...	...
71·5	— 3·93	2·15	— 0·92	13·08	—11·78	...	24·59	...
71·0	— 7·75	...	— 4·76	9·13	...	5·88	...	...
70·5	—11·43	— 5·88	— 8·32	...	—18·91	...	19·27	—26·
70·0	—14·99	...	—11·68	2·25	...	— 1·81	...	—29·
69·5	—18·11	—12·82	...	— 0·70	—24·91	— 5·35	14·12	—31·
69·0	—21·09	—16·02	—17·63	...	—27·70	— 8·58	...	—34·
68·5	—23·74	...	...	— 6·73	—30·04	—11·70	9·10	—37·
68·0	—26·26	—21·73	—22·83	— 9·35	—32·30	...	...	...
67·5	—28·60	...	—25·31	—11·87	—34·40	—17·39	4·10	...
67·0	—30·84	—26·80	—27·49	...	—36·40	—20·03	...	...
66·5	—33·00	...	—29·60	—16·47	—38·45	—22·49	— 0·82	...
66·0	—34·96	—31·36	—31·54	—18·68	...	—24·87	...	...
65·5	—36·84	—33·47	—33·54	—20·68	—41·84	—27·08	— 5·74	...
65·0	...	—35·50	—35·36	—22·70	...	—29·27	...	...
64·5	...	—37·42	...	—24·78	...	—31·25	—10·50	...
64·0	...	—39·15	...	—26·40	...	—33·26	...	...
63·5	...	—40·88	...	—28·00	...	—35·06	—15·33	...
63·0	...	—42·53	...	—29·45	...	—36·90	and other lower readings	...

To face p. 36.

position 71·0, and could, if necessary, be found by interpolation. As, however, the ratio $\log_{10} \frac{\theta_1}{-\theta_2}\Big/\mathrm{L}$ would be constant everywhere if emissivity were negligible, the error is inappreciable, and interpolation is avoided by taking as the zero the nearest reading to it.

(*d*) *Calculation of* Ep/KA *and Verification of Theoretical Formulæ.*—The value of Ep/KA is not required in calculating the thermal conductivity, but additional support is given to the method by determining it, and thus finding the values of λ and μ in equations (1) and (3) of section 2. These equations can then be verified.

Applying to the results of experiment VII., in which the mercury was stationary and heated at the top by ether vapour. the well-known formula :—

$$\mu\frac{\mathrm{L}}{2}=\log\ (n+\sqrt{n^2-1}),$$

where $2n$ is the ratio $\frac{\theta_1+\theta_3}{\theta_2}$, θ_1, θ_2 and θ_3 being successive temperatures separated by the distance L/2, we obtain a mean value

$$\sqrt{\mathrm{E}p/\mathrm{KA}}=\mu=0{\cdot}0598(7)=0{\cdot}06 \text{ very approximately.}$$

The readings of experiment VII. will now be found to agree to a high accuracy with those derived from the formula

$$\theta=80{\cdot}1 \sinh 0{\cdot}06x.$$

Similarly, the equation

$$\theta=\mathrm{M}e^{\frac{ms}{2\mathrm{KA}}x}\sinh x\sqrt{\left(\frac{ms}{2\mathrm{KA}}\right)^2+\frac{\mathrm{E}p}{\mathrm{KA}}},$$

which holds for flows, can easily be tested ; for

$$\frac{ms}{2\mathrm{KA}}=\log_{10}\frac{\theta_1}{-\theta_2}\Big/\mathrm{L}\times 2{\cdot}3026.$$

Applying this to experiment XI., already taken as a typical case, we obtain

$$\frac{ms}{2\mathrm{KA}}=0{\cdot}06104,$$

$$\lambda=\sqrt{\left(\frac{ms}{2\mathrm{KA}}\right)^2+\frac{\mathrm{E}p}{\mathrm{KA}}}=0{\cdot}0856\,;$$

whence $\theta=\mathrm{M}e^{x\times 0{\cdot}061}\sinh\ (x\times 0{\cdot}0856),$

which, if M be found and taken as 81·4, agrees very well with the distribution of temperature as found by experiment. The value of Ep/KA as found from a statical experiment is thus satisfactory when applied to determine λ in dynamical experiments.

(*e*) *Summary of Results of 10 Consecutive Experiments under Flow.*—Table III. is a summary of the deductions made from the sequence of temperatures already given in Table I.

TABLE III.

Magnitude of flow in grammes per 15 mins.	Temperature in degrees Centigrade.	$\text{Log}_{10}\frac{\theta_1}{\theta_2}/\text{L}.$	$\frac{ms}{2\text{KA}}$	λ.	K.
1,296	14·0	0·03139	0·07228	0·094	0·02004
1,159	15·0	0·02778	0·06397	0·088	0·02025
1,199	15·0	0·02898	0·06673	0·090	0·02009
1,003	15·0	0·02449	0·05639	0·082	0·01988
1,422	15·5	0·03420	0·07875	0·099	0·02019
1,079	15·8	0·02592	0·05968	0·085	0·02021
1,099	17·5	0·02651	0·06104	0·086	0·02013
871	17·3	0·02098	0·04831	0·077	0·02015
1,301	15·5	0·03177	0·07315	0·095	0·01988
1,361	15·5	0·03283	0·07559	0·097	0·02013
			Mean....................		0·020095

Thus the result of all experiments is to obtain at 15·5°C. a value for the thermal conductivity of mercury 0·0201 c.g.s. unit.

6. DISCUSSION OF THE METHOD AND THE RESULTS.

The horizontal nature of the isothermals, the constancy of the value of K as deduced from flows of widely different magnitude and the verification of the theoretical formulæ have given the author some confidence in the method of this Paper, though the value he obtains is higher than that usually accepted. How widely the values obtained by other experimenters on this problem differ is seen from the following table :—

TABLE IV.

Experimenter.	Method.	Temperature.	Value of K.
H. F. Weber, 1880	Flat plate..................	17°C.	0·0162
Angström, 1864 ...	Periodic heat flow	50°C.	0·0177
Berget, 1888.........	Guard ring	0°-100°C.	0·02015
R. Weber, 1903 ...	Electrical measurement of heat.....................	0°-34°C.	0·0197

The author's results thus differ widely from those of H. F. Weber and Angström, but are in close agreement with those of Berget and R. Weber. It should be pointed out that objections have been raised to Berget's method (*a*) on account of the uncertainty of the area from which the heat is collected, (*b*) on account of supposed convection currents. The first objection may be serious, but a study of the thermal effect of flows has convinced the author that the second objection is of little moment. For, in the first place, in Berget's inner tube, protected by a guard ring, the isothermals are almost sure to be horizontal, in which case the algebraic effect of convection currents as heat transmitters can easily be shown to be zero; in the second place, it can be shown that in the case of a column of fluid heated at the top natural convection currents will aid rather than hinder the attainment of horizontal isothermals.

Owing to the desire to rotate the thermo-electric carrier tube the lower temperature source was possessed of considerable width; a large quantity of mercury was thus needed for the research. The temperatures of the two constant heat sources—viz., 0°C. and 100°C.—were well suited, owing to the effect of the upward flow, for determining the thermal conductivity over a small range of temperature in the neighbourhood of 15°C. For higher temperatures a thermostatic enclosure would be necessary as well as a higher temperature source.

The essential feature of the method is the simplicity of the calorimetry depending on the well-known value of the specific heat of mercury. Emissivity loss is satisfactorily dealt with, and the range of temperature explored is small. Moreover, only relative, not absolute, temperature differences need be measured.

An interesting problem would be an investigation of the nature of the flow taking into account viscosity and density differences due to temperature. It has, however, been shown elsewhere by the author that as long as the isothermals are horizontal the nature of the flow does not affect the results of these experiments.

This research was carried out in the physical laboratories at Birkbeck College, and the author has been stimulated by the interest shown throughout its development by Dr. A. Griffiths, head of the department.

ABSTRACT.

The Paper gave an account of the determination of the thermal conductivity of mercury at the ordinary temperature of the room by the impressed velocity method first described by the author in the "Proceedings" of this Society, Vol. XXII., 1910. The mercury is contained within a vacuum-jacketed syphon tube about 17 sq. cm. in cross-section, which is heated at the top and maintained ice-cold at its lower extremity. The distribution of temperature down this tube is determined with the aid of a single thermo-junction, while the mercury is at the same time flowing uniformly up the tube at speeds ranging from 870 to 1,420 grammes per 15 minutes. The calorimetry is thus essentially continuous flow calorimetry within the temperature gradient, the quantity of heat passing down the mercury being controlled and measured by the flow of liquid. The wires forming the thermo-elements—viz., iron and constantan—were contained within a vertical tube of 6 mm. external diameter, through the glass of which the wires were fused; this carrier tube lay within the vacuum vessel. The thermo-junction could be raised or lowered to the desired extent by a cathetometer, or could be rotated in a horizontal plane. The isothermals were found to be remarkably horizontal.

If θ_1 is the temperature at a distance L/2 above the isothermal surface at a temperature equal to that of the enclosure, and θ_2 the temperature at an equal distance below, then $\log_e \theta_1/-\theta_2 = ms\mathrm{L}/2\mathrm{KA}$ where s is the specific heat of mercury, K its thermal conductivity, A the cross-section of the tube and m the mass of mercury crossing a section per second. Using this relationship a mean value of 0·0201 c.g.s. units at 15·5°C. is obtained for K.

The advantages of the method lie in the simplicity of the calorimetry and in the fact that temperature ratios rather than temperature differences are required.

DISCUSSION.

Dr. C. Chree said there might be some doubt as to the exactitude with which the mathematical formulæ represented the physical facts, but the method seemed promising for determining the change with temperature of the thermal conductivity, and he inquired whether investigations had been directed towards that end. As the cross-section of the vessel appeared in the formulæ, it would give increased confidence if vessels of different section were tried and found to give accordant results.

Dr. J. A. Harker asked if the author was satisfied as to the distribution of the flow at different parts of the cross-section, and wished to know how the results compared with those of other observers.

Mr. F. E. Smith said that it appeared to him that the practice of Mr. Nettleton did not exactly tally with the conditions imposed by theory. Why did Mr. Nettleton assume that the temperature of some water in a glass tube in the proximity of the apparatus was the same as that of the outer wall of the vacuum jacket? It appeared from the data that it was not, and this being so an error was introduced, inasmuch as the isothermal taken as the "zero isothermal" was not so. If the assumed "zero isothermal" was appreciably higher than the true one the deduced thermal conductivity would be too high. Again, theory imposed a condition on the outer wall of the vacuum jacket; it must be of uniform temperature. Was this so in practice? One end of the jacket was practically at 0°C. and the other at 100°C.; there must be a temperature gradient. He would suggest that the outer wall of the vacuum jacket be cooled with circulating water in which the second thermo-junction was immersed. The beauty of Mr. Nettleton's method was apparent to all, and he trusted he would continue his experiments.

Dr. A. RUSSELL regretted that the author, following the usual custom, called the law for the cooling of a body by radiation Newton's law, and the coefficient the Newtonian coefficient of emissivity. Newton considered the case of a block of iron being cooled by a current of air, so that the heat lost by radiation was very small compared with that lost by convection. In this case he found that the heat lost by the iron was very approximately proportional to the difference of temperature between the air and the iron. This law had been verified recently up to differences of temperature between the cooling body and the air as high as 200°C. and 300°C. In the author's experiment, however, the tube had a vacuum jacket, so that the heat lost by convection was quite negligible. As the difference of temperature was, comparatively speaking, small, we saw by Stefan's law that the author's assumption was justifiable. In the speaker's opinion the real "Newton's law of cooling" was of great importance in practical work, and he thought that teachers and the writers of text-books ought to lay greater stress on it.

Dr. W. H. ECCLES called attention to the fact that at first sight the method would be thought to be incapable of high accuracy, because of the form of the expression for K, the conductivity. This expression had as its denominator what was practically the difference between two nearly equal magnitudes, and these magnitudes were themselves differences. That was to say, the denominator was of the nature of a second differential coefficient of an experimental curve. That such consistent results were obtained indicated remarkable precision of experiment.

Prof. LEES asked how small a difference of temperature throughout the cross-section would have been detected by the thermo-couple.

Dr. GRIFFITHS, in reply, stated that, as shown in the earlier Paper, the distribution of the flow was not important, so long as the isothermals were plane.

The AUTHOR communicated the following: In reply to the chairman, Prof. Lees, it should be stated that a difference of temperature of 0·05°C. over a horizontal surface would easily have been detected in any position of the cathetometer. In the neighbourhood of the "zero isothermal" all resistances, except that of the galvanometer and wires, could be removed and a more delicate test made. It is safe to say that in this neighbourhood a difference of 0·02°C. would have been detected easily. Thus the isothermals are remarkably horizontal, the result far exceeding the author's expectations. Mr. F. E. Smith has raised an important point in his remarks about the "cold junction" at the temperature of the enclosure. The cold junction could not be inside the vacuum, as iron and constantan cannot be fused through glass and a vacuum of high standard maintained. An attempt to surround an earlier vacuum vessel with a glass water jacket (it is necessary to be able to see through the vacuum when adjusting the carrier-tube) resulted in its collapse, and no risks were taken with the present vessel, which is valuable, as at least 10 attempts at making a third have failed. The "cold junction" was maintained in a tube under water just outside the vacuum vessel to protect it from air draughts, but on holding it in the air just outside the vacuum vessel no serious temperature difference was recorded. The vacuum, of course, was of "thermos" or X-ray standard. When the "warm junction" was in such a position as to produce no electric current the zero-isothermal was located. The author cannot doubt that the temperature of the cold junction was rightly that of the enclosure; for it stood the double test of Newton's law holding with respect to it in a statical experiment, as well as the constancy of the ratio $\log \frac{\theta_1}{-\theta_2}/\mathrm{L}$ in many dynamical experiments. The height of the zero-isothermal varies in every experiment, being higher the faster the flow, as can be seen in the actual temperature curves given in the Paper previously alluded to. Under no flow the zero was very low, being near the lower extremity of the vacuum-jacketed portion of the experimental tube—too low, in fact, to allow of the test $\theta_1/-\theta_2=1$ over any considerable range. The range of temperature between the extremities of the vacuum would be of the order 30°C.—35°C

IV. *Polarisation and Energy Loss in Dielectrics.* *By* A. W. Ashton, *D.Sc., M.I.E.E., Battersea Polytechnic, Battersea, S.W.*

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§ 1. *Introduction.*—The object of this Paper is to discuss the relations which should exist between the coefficients in Pellat's equation (as modified by Schweidler) giving the displacement in a viscous dielectric as a function of the time of charge and the P.D.

Pellat* showed that (at least qualitatively) the displacement in a dielectric can be represented by

$$D_t = KE_0 + \varepsilon KE_0(1 - e^{-\alpha t}),$$

where D_t=displacement at time t since P.D. E_0 was applied, α, K and ε are constants, K being the instantaneous capacity of the condenser and α the reciprocal of the time constant of the viscous displacement, while ε is a coefficient such that displacement after infinite time of charge is equal to $KE_0(1+\varepsilon)$.

Pellat recognised that it would be necessary in some cases to use several terms having different values for α in order to represent quantitatively the results obtained from dielectrics.

It was pointed out by Schweidler† that by modifying Pellat's equation into the form

$$D_t = KE_0 + KE_0 \Sigma\varepsilon(1 - e^{-\alpha t})$$

the charging current for constant voltage becomes for $t > 0$

$$i_t = KE_0 \Sigma \alpha\varepsilon e^{-\alpha t},$$

and in the series of exponential terms α and ε can be so chosen as to make

$$\Sigma \alpha\varepsilon e^{-\alpha t} = \beta t^{-n}.$$

This is necessitated by the results of experiments in which condensers have been charged for a considerable time at a constant voltage. The charging current for $t > 30$ seconds has been shown in a large number of cases to be of the form

$$i_t = i_a + \beta KE_0 t^{-n},$$

where $n < 1$, and it has generally been considered that i_a is a true conduction current whilst the recoverable energy is due to the term $\beta KE_0 t^{-n}$.

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* "Annales de Chimie et de Physique," Ser. 7. Vol. XVIII., p. 150, 1899.

† "Annalen der Physik," Vol. XXIV., p. 711, 1907.

§ 2. *Relations between α and ε for any given Value of the Exponent n in the Equation for Polarisation Current.*—If we consider the above equation for polarisation current it can be shown that, in order that the current may be proportional to t^{-n}, the variations of α and ε must be such that the product $\alpha\varepsilon$ is proportional to α^n.

In a physical sense $K\varepsilon$ may be considered to be the viscous displacement due to unit P.D. (*i.e.*, the viscous capacity) corresponding to all molecules of the dielectric having the same displacement time constant.

Let $$C_1 = \Sigma\alpha\varepsilon e^{-\alpha t_1} = \beta t_1^{-n}$$
and let $$C_2 = \Sigma\alpha\varepsilon e^{-\alpha t_2} = \beta t_2^{-n}.$$

In order to make $C_1 = kC_2$, where k is any positive number, we must have $t_1^{-n} = kt_2^{-n}$, *i.e.*, $t_1 = k^{-1/n}t_2$.

Although α varies continuously we can choose the values of α in the series so that the values of $\alpha\varepsilon$ are in geometrical progression and write

$$C_1 = \alpha_1\varepsilon_1 e^{-\alpha_1 t_1} + \alpha_2\varepsilon_2 e^{-\alpha_2 t_1} + \ldots\ldots\ldots\ldots + \alpha_r\varepsilon_r e^{-\alpha_r t_1}.$$

where $\alpha_2\varepsilon_2 = k\alpha_1\varepsilon_1$, $\alpha_3\varepsilon_3 = k\alpha_2\varepsilon_2$, &c.

In this case $k\varepsilon_m$ is the total capacity of that group of molecules whose time constants have a mean value $1/\alpha_m$, and which fall within certain limits as determined by the values of α in the terms on either side.

Now $$C_2 = \frac{1}{k}\{\alpha_2\varepsilon_2 e^{-\alpha_1 t_2} + \alpha_3\varepsilon_3 e^{-\alpha_2 t_2} + \ldots + \alpha_{r+1}\varepsilon_{r+1} e^{-\alpha r t_2}\},$$

and since, when $1/\alpha$ is either very small or very large compared with t, $\alpha\varepsilon e^{-\alpha t}$ becomes negligible, we can write $C_2 = \frac{C_1}{k}$, provided $\alpha_2 t_1 = \alpha_1 t_2$, $\alpha_3 t_1 = \alpha_2 t_2$, $\alpha_4 t_1 = \alpha_3 t_2$, &c., &c., and the first and last terms of the series are negligibly small.

Since $t_1 = k^{-1/n}t_2$, in order to make $\alpha_2 t_1 = \alpha_1 t_2$, $\alpha_3 t_1 = \alpha_2 t_2$, &c., we must have $\alpha_2 = k^{1/n}\alpha_1$, $\alpha_3 = k^{1/n}\alpha_2$, &c., and therefore $\alpha_2^n = k\alpha_1^n$, $\alpha_3^n = k\alpha_2^n$, &c., from which it follows that since $\alpha_2\varepsilon_2 = k\alpha_1\varepsilon_1$, $\alpha_3\varepsilon_3 = k\alpha_2\varepsilon_2$, &c., therefore the values of α and ε must be so chosen that $\alpha\varepsilon$ shall be proportional to α^n.

From this it follows further that ε is proportional to $(1/\alpha)^{1-n}$ and we must have the following law :—

Where the polarisation current of a dielectric is proportional to t^{-n} the viscous capacity of any group of molecules having the same time constant of displacement is proportional to the $(1-n)$th power of the time constant.

§ 3. *Energy Loss and Variation of Capacity with an Alternating E.M.F. as determined from the Equations of Hopkinson and Wilson.*—Equations for the displacement in a viscous dielectric under the action of an alternating E.M.F. have been given by Hopkinson and Wilson* on the assumption that the charging current is proportional to t^{-n}, where $n<1$. From these equations the apparent capacity is given by

$$C=K+p^{n-1}\beta K\Gamma(1-n)\cos(1-n)\frac{\pi}{2}$$

and the alternating-current conductivity by

$$S=S_0+p^n\beta K\Gamma(1-n)\sin(1-n)\frac{\pi}{2},$$

where S_0 is the true conductivity under a steady E.M.F. and $p=2\pi f$, where f=frequency of the applied E.M.F.

For all frequencies greater than about one per second S_0 is negligible compared with S, the energy loss being therefore proportional to f^n and the viscous capacity proportional to f^{n-1}.

Now the apparent conductivity for a steady E.M.F. is

$$S''=\beta Kt^{-n}$$

and the ratio of the alternating-current conductivity to the apparent conductivity under a steady E.M.F. is

$$r=p^nt^n\sin(1-n)\frac{\pi}{2}\Gamma(1-n)$$

$$=\frac{p^nt^n\pi}{2\Gamma(n)\cos(1-n)\frac{\pi}{2}}.$$

This ratio is of considerable importance in connection with cable testing in which it has been the practice to measure the apparent insulation resistance after one minute's electrification in order to detect faults in manufacture.

The above expression for alternating-current conductivity becomes negative when $n>2$ and this is also the case with regard to the viscous capacity when $n>1$. The variations of energy loss and capacity with frequency can be shown to be true within the above limits for n by investigating the series obtained on the assumption that the value of ε and α are such that $\alpha\varepsilon$ is proportional to α^n.

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* "Phil. Trans.," Vol. 189A., 1897.

§ 4. *Variation of Energy Loss with Frequency as determined from Pellat's Theory.*—The total alternating-current conductivity of the dielectric due to viscous polarisation is

$$S=K\Sigma\frac{p^2\varepsilon\alpha}{p^2+\alpha^2}.$$

If we consider two terms of the series, α_1, ε_1 and α_2, ε_2 such that $\alpha_2=k\alpha_1$, then $\alpha_2\varepsilon_2=k^n\alpha_1\varepsilon_1$ and the conductivity at frequency $p_2/2\pi$ for the group α_2, ε_2 is

$$S'_2=\frac{Kp_2{}^2\alpha_2\varepsilon_2}{p_2{}^2+\alpha_2{}^2}.$$

Now, if S'_1=conductivity at frequency $\frac{p_1}{2\pi}$ such that $p_2=kp_1$ for the group α_1, ε_1, then it follows that

$$S'_2=\frac{Kp_1{}^2\alpha_1\varepsilon_1k^{n+2}}{k^2(p_1{}^2+\alpha_1{}^2)}=k^nS'_1.$$

Therefore in the series for the total conductivity of the dielectric at frequency $p_2/2\pi$, the terms are each k^n times the corresponding terms in the series for conductivity at k^{-1} times the frequency, and provided the terms of the two series vanish in the limit when α is indefinitely increased or diminished we can write S proportional to p^n.

Now it can be shown that S′ is a maximum for a given frequency in the case of that term of the series for which

$$\alpha=p\sqrt{\frac{n}{2-n}}.$$

For values of n lying between 0·5 and 1·5 the maximum energy loss occurs in those groups for which α does not greatly differ from p, while for values of α much greater than this $S'=\frac{Kp^2\varepsilon}{\alpha}$. Further, since $\frac{\varepsilon}{\alpha}$ is proportional to α^{n-2} the terms in the series must vanish in the limit as α increases provided n is <2. For values of α much smaller than p, $S'=K\varepsilon\alpha$, where $\varepsilon\alpha$ is proportional to α^n and the terms in the series vanish in the limit as α is diminished provided n is positive. It appears therefore that for all positive values of $n<2$ the alternating-current conductivity is proportional to the nth power of the frequency.

§5. *Variation of Viscous Capacity with Frequency as determined from Pellat's Theory.*—The capacity under the application of an alternating E.M.F. is given by $C=K\left(1+\Sigma\frac{\alpha^2\varepsilon}{p^2+\alpha^2}\right)$. If, as before, α_1, ε_1 and α_2, ε_2 are values of α and ε such that $\alpha_2=k\alpha_1$, then it follows that $\alpha_2\varepsilon_2=k^n\alpha_1\varepsilon_1$. The viscous capacity for the group α_2, ε_2 at a frequency $p_2/2\pi$ is

$$\begin{aligned}C'_2&=\frac{K\alpha_2{}^2\varepsilon_2}{p_2{}^2+\alpha_2{}^2}\\&=\frac{Kk^{n+1}\alpha_1{}^2\varepsilon_1}{k^2(p_1{}^2+\alpha_1{}^2)}\\&=k^{n-1}C'_1,\end{aligned}$$

where C'_1 is the viscous capacity for the group α_1, ε_1, at frequency $\frac{p_1}{2\pi}=\frac{p_2}{2\pi k}$. Provided the terms of the two series vanish in the limit as α either increases or decreases the viscous capacity will be proportional to the $(n-1)$th power of the frequency.

Now it can be shown that $\frac{\alpha^2\varepsilon}{p^2+\alpha^2}$ is a maximum for $\alpha=p\sqrt{\frac{1+n}{1-n}}$, and, provided n is not approaching unity, there is a well-defined maximum value for which α is in the neighbourhood of p.

As α diminishes so that α^2 becomes negligible compared with p^2 the capacity of each group $=\frac{K\alpha^2\varepsilon}{p^2}$, which is proportional to α^{n+1}, and the terms of the series vanish in the limit for all positive values of n. On the other hand, as α increases so that p^2 is negligible compared with α^2 the capcity of each group becomes $=K\varepsilon$, which is proportional to α^{n-1}, and the terms of the series only vanish in the limit as α increases provided $n<1$. The viscous capacity will therefore be proportional to f^{n-1} provided $n<1$.

It will be noticed that for a given frequency the groups of molecules having the greatest influence on the capacity and conductivity may have very different time constants. Thus the ratio of the value of α for maximum conductivity to that for maximum viscous capacity is given by $d=\sqrt{\frac{n(1-n)}{(2-n)(1+n)}}$. The value of d is approximately one-third when n lies between 0·1 and 0·9 but increases rapidly as n approaches unity.

§ 6. *Variation in Capacity when n is Unity.*—The variation in capacity with change of frequency can easily be determined for the case in which $n=1$. In this case ε is constant and α alone varies. The terms in the series for capacity are all equal until α becomes comparable with p. From this point the terms in the series diminish to zero, and this part of the series is the same for all values of p. The number of equal terms becomes greater as p diminishes. If α_1, α_2 be values of α and p_1, p_2 values of p such that $\frac{\alpha_1}{p_1}=\frac{\alpha_2}{p_2}$, then the viscous capacity C_1 for frequency $p_1/2\pi$ differs from the capacity C_2 for frequency $p_2/2\pi$ by $K\Sigma_{\alpha_1}^{\alpha_2}\varepsilon$.

We may therefore write, if $p_2>p_1$,

$$C_1=C_2+C_0 \log_{10}\frac{p_2}{p_1},$$

where

$$C_0=K\Sigma_{\alpha}^{10\alpha}\varepsilon.$$

From this it follows that for frequencies increasing in geometrical progression the capacity decreases in arithmetical progression.

A similar formula can be shown to represent the variation of capacity with time under the action of a steady E.M.F.

The above properties of the dielectrics should be true whether n is a constant over the whole range or not, since, except in cases where n is approaching its limiting value for the particular property considered (such as $n=1$ for capacity variation), the range of values of α having most effect in any given case will be restricted within limits closely corresponding to the frequency or the time for which the variation is being investigated.

§ 7. *Experimental Evidence of Theory.*—A large amount of valuable experimental research on dielectrics has been recently carried out by various investigators and it is proposed to examine some of the results obtained in order to test the above conclusions.

Addenbrooke* has investigated the energy loss in gutta-percha for comparatively slow cycles and has expressed his results for each frequency as a multiple of the apparent loss with a steady E.M.F. after two minutes' application. Except for frequencies of less than 2 cycles per second the ratio of losses can be shown to be fairly accurately represented by

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* *See* "The Electrician," Vol. LXX., p. 673.

$r=93f^{0\cdot 79}$, where f=periods per second. Table I. gives observed and calculated values of r.

Table I.—*Ratio of Loss in Gutta Percha under an Alternating E.M.F. to that under a Steady E.M.F. as given by Addenbrooke.*

f.	r (observed).	r (calculated).	Error %.
0·2	20	25·5	+27·0
0·4	38	44·0	+16·0
1·0	89	93·0	+ 4·5
2·0	160	160·0	0
4·0	276	281·0	+ 1·8
8·0	480	482·4	+ 0·5
16·0	830	829·0	− 0·1
32·0	1,440	1,440·0	0
42·0	1,830	1,785·0	− 2·5

A large number of dielectrics have been tested by Fleming and Dyke* over a wide range of temperature for alternating-current conductivity and capacity at telephonic frequencies. From these experiments they conclude that for most dielectrics the alternating-current conductivity can be represented by a formula $S=A+Bf$, where f is frequency and A and B are constants, of which, however, A is very small for most good insulators. Their results can also be shown to be closely represented by an equation of the form $S=Qf^n$, where Q is a constant, the case in which $A=0$ corresponding to $n=1$. The value of n varies from 0·4 to 1·4 but is nearly unity for a considerable number of the dielectrics tested. Table II. shows the values of n for celluloid at temperatures varying from −184°C. to 80°C. taken from the same Paper, and, except at 15°C. and 60°C., the calculated values do not differ more than 2 per cent. from the observed values.

Table II.—*Values of n determined from Fleming and Dyke's Tests on Celluloid.*

n.	°C.	n.	°C.	n.	°C.
0·85	−184°	1·22	−11°	0·56	60°
0·90	−140°	1·15	2·5°	0·49	68°
1·02	−120°	1·05	15°	0·44	80°
0·99	− 75°	0·71	30°	...	...
1·20	− 38°	0·59	51°	...	...

Curtis† in connection with a series of tests for dielectric loss and capacity variation in mica condensers, has discussed the question as to the agreement between his results and Schweidler's modification of Pellat's theory and appears to have com to the conclusion that the agreement is not satisfactory. In at least one case, however, agreement between his results and the theory can be shown to be remarkably close.

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* *See* "Journal" Inst. Elec. Eng., Vol. XLIX., p. 323,1912.

† *See* "Bulletin" of Washington Bureau of Standards, Vol. VI., p. 431.

Columns 1, 2 and 5, Table III., show the results Curtis obtained where $\varphi=90-\theta$ for power factor $\cos\theta$. ΔC is variation in capacity (parts in 100,000). S is the conductivity for alternating currents for a condenser of 1 microfarad capacity. The fourth column gives the conductivity calculated from $S=5{\cdot}334\times10^{-9}p^{n}$, where $n=0{\cdot}654$, and the sixth column the viscous capacity calculated from $\Delta C=853{\cdot}3p^{n-1}$.

TABLE III.—*Conductivity and Capacity of Mica Condenser* (*Curtis*).

f.	ϕ.	$S\times10^{6}$ (observed).	$S\times10^{6}$ (calculated).	ΔC (parts in 100,000 observed).	ΔC (parts in 100,000 calculated).
1,200	50″	1·827	1·8317	7	38·86
100	2′ 0″	0·3654	0·3606	60	91·83
50	2′ 30″	0·2284	0·2291	85	116·7

The increase in capacity from $f=1{,}200$ to $f=100$ is 53 observed and 52·97 calculated, while from $f=100$ to $f=50$ the corresponding values are 25 and 24·87.

The agreement is even more remarkable with regard to the values of βK calculated from the conductivity and capacity measurements. For the former we have

$$S=\frac{\beta K\pi p^{n}}{2\Gamma(n)\cos(1-n)\frac{\pi}{2}}=5{\cdot}334\times10^{-9}p^{n},$$

where $n=0{\cdot}654$.

From this $\beta K=4{\cdot}005\times10^{-9}$.

From the capacity variation

$$\Delta C=8{\cdot}533\times10^{-9}p^{n-1}$$

in practical electromagnetic units, and from this $\beta K=3{\cdot}87\times10^{-9}$, which agrees with the former value within 3½ per cent.

H. A. Wilson* has carried out an investigation of the change in capacity due to absorption under the action of a steady E.M.F., and has shown that the capacity after T minutes bears a ratio $1+B\log(1+p'T)$ to the capacity after 1 minute's charge.

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* *See* "Proc." Roy. Soc., Vol. 82A., p. 409, 1909.

In the case of the ebonite condenser it can be shown that the capacity at 30°C. is very closely represented by

$$C=0{\cdot}4545+0{\cdot}3618\,T^{0{\cdot}098},$$

the maximum deviation of the calculated values from the values observed being 0·15 per cent. From the above equation $n=0{\cdot}902$, which is in agreement with values of 0·9 to 1·0 given by Fleming and Dyke's results on ebonite.*

It is interesting to note that equations for conductivity and capacity variation under an alternating E.M.F. can be readily obtained from the above equation.

For t in seconds we have

$$C=0{\cdot}4545\times10^{-6}+0{\cdot}2421\times10^{-6}\,t^{0{\cdot}098}$$

in electromagnetic units, from which it follows that

$$\beta K=2{\cdot}373\times10^{-8}.$$

The alternating-current conductivity is given by

$$S=3{\cdot}534\times10^{-8}\,p^{0{\cdot}902}.$$

The capacity under alternating E.M.F. is given by

$$C=0{\cdot}4545\times10^{-6}+2{\cdot}278\times10^{-7}\,p^{-0{\cdot}098}.$$

The above discussion of results obtained on various dielectrics shows the possibility of correlating the various absorption phenomena of a dielectric. Before coming to any very definite conclusion, however, it is proposed to carry out a series of tests on various dielectrics over a wide range of frequency, as well as with steady E.M.F.s, in order to determine to what extent n is constant for a given dielectric at a definite temperature.

PELLAT'S THEORY.†

The following brief résumé of Pellat's theory of "true polarisation" differs mainly from the original treatment in that the condenser is assumed to have only one layer of dielectric between the plates instead of also having two layers of air.

Let λ=specific inductive capacity of the dielectric,
φ=electric intensity,
σ=surface density on the condenser plates.

Then $$\varphi=\frac{4\pi}{\lambda}(\sigma-j), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where j is called by Pellat the true polarisation. At the end of infinite time j takes a final value, J. It is assumed that the rate of variation of j is proportional to the difference between its present and final values—that is,

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* *See* "Journal" Inst. Elec. Eng., *loc. cit.*

† *See* Pellat, "Annales de Chemie et de Physique," Vol. XVIII., p. 150. Also Pellat and Beaulard, "Comptes Rendus," Vol. CXXX., p. 1457.

$$\frac{dj}{dt}=a(J-j). \qquad (2)$$

From which it follows that

$$j=J(1-e^{-at}). \qquad (3)$$

Pellat justifies the assumption made in equation (2) by proving from it Curie's Principle of Superposition.

The final polarisation J is a function of the final electric intensity Φ, and we may write $J=h\Phi$, where h is a constant for weak fields but a function of Φ for strong fields.

If a steady E.M.F., E_0, be applied to the dielectric we have

$$J=h\frac{E_0}{c}, \qquad (4)$$

where c=thickness of dielectric, and from (1) and (3)

$$\sigma=\frac{\lambda E_0}{4\pi c}+cJ(1-e^{-at}). \qquad (5)$$

Put $K=\frac{\lambda A}{4\pi c}$, where A=area of each condenser plate, then the displacement (D_t) at time t is given by

$$D_t=KE_0+AhE_0(1-e^{-at}) \qquad (6)$$

and

$$D=KE_0+\varepsilon KE_0(1-e^{-at}), \qquad (7)$$

where

$$\varepsilon=\frac{Ah}{K}.$$

The current is therefore given by

$$i=KE_0\varepsilon ae^{-at}. \qquad (8)$$

The capacity and the energy loss under the application of an alternating E.M.F. were deduced by Pellat and Beaulard on the assumption that h is a constant.

Let $e=E_m \sin pt$ be the applied E.M.F. From equation (2)

$$\frac{dj}{dt}+aj=\frac{ah}{c}E_m \sin pt. \qquad (9)$$

The solution of equation (9) is $j=B\sin(pt+\theta)$, where

$$B=\frac{E_m ha}{c\sqrt{p^2+a^2}}$$

and

$$\cos\theta=\frac{a}{\sqrt{p^2+a^2}}, \quad \sin\theta=\frac{-p}{\sqrt{p^2+a^2}}. \qquad (10)$$

From (1), (4) and (10) we have

$$\sigma=\frac{\lambda \mathrm{E}_m \sin pt}{4\pi c}+\mathrm{B}\sin(pt+\theta).$$

Therefore $i=\mathrm{A}\frac{d\sigma}{dt}=p\mathrm{KE}_m\cos pt+\frac{p\mathrm{KE}\varepsilon_m a}{\sqrt{p^2+a^2}}\cos(pt+\theta)$;

that is, $i=p\mathrm{KE}_m\left\{1+\frac{\varepsilon a^2}{p^2+a^2}\right\}\cos pt+\frac{p^2\mathrm{KE}_m\varepsilon a}{p^2+a^2}\sin pt.$

The capacity under an alternating E.M.F. is

$$\mathrm{C}=\mathrm{K}\left(1+\frac{\varepsilon a^2}{p^2+a^2}\right)$$

and the rate at which energy is lost is given by

$$w=\frac{p^2\mathrm{KE}_m{}^2\varepsilon a}{2(p^2+a^2)}.$$

ABSTRACT.

The Paper discusses the relations which should exist between the coefficients in Pellat's equation (as modified by Schweidler) giving the displacement in a viscous dielectric as a function of the time of charge and the P.D. It is proved that where the polarisation current of a dielectric under the action of a steady E.M.F. is proportional to t^{-n} (t=time), the viscous capacity of any group of molecules having the same time constant of displacement is proportional to the $(1-n)$th power of the time constant.

Starting with this hypothesis it is proved that under an alternating E.M.F. the energy loss varies as the nth power of the frequency, and the viscous capacity varies as the $(n-1)$th power. An equation is also obtained giving the variation in capacity for the particular case in which $n=1$.

Experimental results published by various investigators are given, and are shown to be not inconsistent with Pellat's modified theory. A brief résumé of Pellat's Theory is also given.

DISCUSSION.

Dr. Russell said that the author's method was ingenious and that the formula deduced for the dielectric current agreed well with many experimental results. He thought, however, that Pellat's theory was not sound and that von Schweidler's modification of it was too elaborate to be of much practical use. He pointed out that Dr. Grover had proved that, from the mathematical point of view, Hopkinson's and von Schweidler's theories were identical. He considered that Fleming and Dyke's experimental results could not be ignored, and that none of the theories advanced up to the present explained them satisfactorily. Very different formulæ gave results which were practically identical over wide ranges, and so they were of doubtful help in enabling us to discover the causes of the phenomena. He considered that the nomenclature of dielectric theory stood badly in need of standardising. For instance, what the author called the displacement was practically the same as Maxwell's electric displacement, but Sir J. J. Thomson and several other writers called it the polarization.

Dr. ECCLES stated that the Paper compelled remark on the artificial nature of Pellat's hypothesis. This hypothesis led to an expression for the displacement current as a sum of exponential terms. Experiment required it to be a power of t, or the sum of several power terms. This required that the exponential terms of Pellat's hypothesis should be infinite in number—which deprived the hypothesis of physical meaning. It would be more logical, therefore, to begin with the power terms as fundamental, and to discard the false start given by Pellat.

Mr. ADDENBROOKE stated that in order to get at the proper theory of experiments the best dielectrics should not be used, as the effects were so minute that the chances of error were great. With poorer dielectrics, the actions were on a greater scale and easier to observe. Using an ellipsoid suspended at 45 deg. to the electric field, he had been able to carry his experiments down to a frequency of 1 cycle in two hours, taking, of course, time readings of current and voltage. Dr. Ashton had not mentioned a Paper by Wagner, which seemed to cover much the same ground.

Prof. C. H. LEES mentioned a later Paper on the subject by Curtis, who had found that Schweidler's equations held if several terms were taken.

The AUTHOR, in reply, agreed with Dr. Russell as to the need for a standard nomenclature with regard to dielectrics, and called attention to the numerous methods of expressing the quantity known as the electric force. With regard to Dr. Russell's objection that the exponent n has not been shown to be constant, except over a limited period of time, some of the author's published results show that n has remained constant for over 24 hours, but it appeared impossible to carry out satisfactory experiments over long periods without adopting stringent precautions for keeping the temperature and the applied voltage absolutely constant. The author did not wish to give the impression that Fleming's and Dyke's results were not of the highest accuracy, but he believed their results were equally consistent with the equations deduced from Pellat's theory as with the equation of Fleming and Dyke. With regard to the criticism of Dr. Eccles and Prof. Lees that a theory involving the representation of results by an infinite series was of little use, the author contended that the theory did not necessarily involve an infinite series. The upper limit of α is fixed by the point at which the capacity ceases to be affected by an increase in frequency. The lower limit is fixed by the time required to fully polarise the dielectric by a steady E.M.F. An infinite series would lead to the displacement being infinite for $t =$ infinity. The great advantage of the theory is the fact that it correlates all the so-called "absorption" phenomena, and if established by experiment it could be used to extend our knowledge of the properties of dielectrics. For instance, it can be shown that if the theory is true the hysteresis loop must be elliptical, and that a third harmonic cannot be introduced by dielectric hysteresis as it can be by magnetic hysteresis. With regard to Mr. Addenbrooke's work on slow cycles, it should be pointed out that at frequencies of the order of one per second the rate of absorption of energy becomes comparable with the rate of dissipation in the dielectric, and according to Pellat's theory the former will depend on the particular point in the cycle at which the E.M.F. was first applied to the condenser.

V. *A Lecture Experiment to Illustrate Ionisation by Collision and to show Thermoluminescence.*

By F. J. Harlow, *A.R.C.S.*, *B.Sc.*, *Assistant Lecturer in Physics, Sir John Cass Technical Institute.*

Read November 14th. Received November 10th.

An experiment to test the effect of providing the electrodeless discharge with a continuous supply of ions from incandescent solids proved to be a suitable one for demonstrating to an audience several points in connection with the production of ionisation by collision, and was thought, therefore to be worth recording.

Experimental Arrangement.

A spherical discharge bulb was constructed, with four leads passing into it carrying two small coils of platinum wire coated with aluminium phosphate and lime respectively. These substances yield when heated abundant supplies of ions, positive from the phosphate and negative from the lime. The platinum wires were placed near the surface of the glass, and in the plane of the coil wound round the bulb for the purpose of producing the electrodeless discharge. The arrangement used to set up the necessary electrical oscillations in this coil was that involving the use of two condensers, described by Sir J. J. Thomson in "Recent Researches." The alternative method of exciting the discharge, which involves the use of one condenser only, was found to be unsuitable on account of the large electrostatic effect which is produced, the ionisation from this masking somewhat the effect to be described.

Effect of Incandescent Lime and Aluminium Phosphate.

The intensity of the electrodeless discharge produced by the above arrangement depends on both the gas pressure and the induced E.M.F. For any given E.M.F. the discharge can be excited over a limited range of pressure only, and within this range, as the pressure is diminished, its intensity increases to a maximum, after which it decreases slightly, then becomes visibly intermittent and finally disappears altogether. By heating up either the lime or the aluminium phosphate to incandescence two noticeable effects are to be observed. First, the intensity of the discharge, provided this is not too great initially, is considerably increased ; and, secondly, the lower limit of the pressure range over which the discharge can be produced is considerably extended.

The first of these effects shows in a striking way the increased ionisation in the gas caused by the collision of ions from the incandescent solid with the molecules of the gas. This increase is not so very noticeable at the higher pressures, but becomes very marked as the pressure is reduced. The colour, too, of the discharge often undergoes a distinct change, similar in character to that which occurs when the pressure is reduced or the E.M.F. increased, which points rather to an increased velocity at collision. The effect of the negative ions from the lime is generally greater than that of the positive ions from the phosphate, unless the latter is heated very strongly, when, according to Garrett,* negative ions are given off in addition.

The second effect—viz., that of increasing the range of the discharge—is, perhaps, the more striking, as, when the pressure is sufficiently low that no discharge can be obtained in the ordinary way, a vivid discharge ensues as soon as either the lime or the phosphate is heated. This is an illustration of the fact that the sparking potential is considerably reduced by the presence of initial ionisation, since the discharge being intermittent is practically a succession of sparks. The extension of the range of the discharge in this way is greater with the negative ions than with the positive, which shows that the latter are less effective than the former in producing ionisation by collision. The precise limits of the range of pressure over which the discharge can be excited depends largely upon the induced E.M.F. The values obtained with one particular arrangement will show the order of the effect of the ionisation from the heated solids. The discharge produced without the assistance of the lime or phosphate commenced at 0·3 mm. pressure, and disappeared at 0·05 mm. When the lime or phosphate was heated no appreciable difference in the pressure at which the discharge started could be detected ; but the lower limit with the phosphate was extended to 0·02 mm., and with the lime to a pressure lower than could be recorded by a McLeod gauge reading to 1/1000 mm.

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* "Proc." Phys. Soc., November, 1910.

An interesting point demonstrated by the experiment is the well-known spreading of the discharge as the pressure is diminished on account of the increased diffusion of the ionisation at the lower pressures. The discharge at the very low pressures completely fills the bulb, whereas at the higher pressures it is just a ring near the surface of the glass. As the pressure is reduced, however, the discharge gets feebler and feebler as the number of molecules in the bulb, and therefore the number of collisions, is gradually reduced.

To make the experiment most effective, the ordinary discharge should be made quite feeble. This can be done most conveniently within certain limits by an adjustment of the spark-length ; but it may be necessary to adjust the number of turns in the oscillation coil. The best effect with the lime is given after it has been heated for some considerable time, for, as Willows and Picton have shown,* the electrical activity of lime is increased enormously by this process. If, however, it be exposed again to air at atmospheric pressure, the activity returns to a very small value. On the other hand, Garrett has, shown† that the activity of aluminium phosphate is decreased by continued heating, assuming eventually a final steady value, so that this is more effective when first set up.

Thermoluminescence.

When the experiment described above was set up it was found that the lime possessed the property of displaying thermoluminescence. A discharge intensified by the phosphate alone was maintained for a few minutes, the lime being kept cold. When this was stopped, and the lime gently warmed by a current much less than that required to raise it to incandescence, a golden yellow luminosity was emitted which persisted for some considerable time, unless the lime was heated above a certain temperature, when the luminosity instantly ceased. The lime had then to be exposed afresh to the discharge before the phenomenon recurred. The experiment can be shown most effectively by switching on a current of sufficient strength to make the lime incandescent. As the lime is warming up it first flashes out intensely the thermoluminescence, then ceases to be luminous, and finally becomes incandescent. The phenomenon of thermoluminescence, which usually occurs with substances containing certain others as impurity, is thought to be due to a soft Röntgen radiation* produced in the discharge at the moment of collision of the ions with the gas molecules. This radiation, called "Entladungstrahlen," by Wiedemann, is considered to produce a chemical compound stable at ordinary temperatures, but which breaks up, however, when warmed, emitting a characteristic luminosity as it does so. The method used for coating the platinum wire with lime was that of putting on a layer of sealing wax and raising to incandescence. The white residue consists largely of lime, and adheres well to the platinum, but contains barium sulphate and other substances as impurities. The thermoluminescence was thought at first to be due to these impurities, but nominally pure lime was found to produce the phenomenon equally well. This case of thermoluminescence does not appear to have been observed before.

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* "Proc." Phys. Soc., June, 1911.

† Loc. cit.

Tests made with the aluminium phosphate showed that it also gave a feeble blue thermoluminescence which, however, could not be obtained after the phosphate had been heated for some time.

I am indebted to Dr. R. S. Willows for his kind interest and helpful suggestions in connection with this experiment.

Note added November 28th.

Since the above Paper was first sent in for publication the peculiarity of lime and aluminium phosphate in exhibiting thermoluminescence has received further attention. As mentioned in the Paper, the phenomenon does not usually occur with pure substances, so that it seemed curious that it should be observed with lime and aluminium phosphate, both of which were thought to be pure. A test was made to see whether or not the lime, in which the phenomenon was most marked, gave any thermoluminescence apart from the platinum. A piece of platinum foil coated with lime was first held near the vivid electric sparks passing across a spark-gap, and then gently warmed in the hot air rising from a small non-luminous gas flame. When the requisite temperature was reached the characteristic luminosity appeared. The experiment was now repeated with lime on a watch glass, but no thermoluminescence could be observed at all. It is thus the combination of the lime and platinum which causes the phenomenon. This observation suggests that there may be a close connection between thermoluminescence and electrical activity. This suggestion receives considerable support from an observation of the effect in the electrodeless discharge bulb. As already mentioned, Willows and Picton have observed that, with continued heating, the electrical activity of lime is considerably increased. Now this was found to be the case with the thermoluminescence. After keeping the lime incandescent for several hours, both its efficiency in the experiment described and the thermoluminescence increased enormously. Experiments on the effects of other forms of radiation have revealed the fact that both ultra-violet light and Röntgen radiation from a bulb are capable of exciting the phenomenon. Other metals besides platinum have also been tried. Lime on nickel, which is also electrically active,* gives thermoluminescence of the same colour ; lime on aluminium gives a blue luminosity similar to that given by aluminium phosphate on platinum, but lime on copper gives no effect at all.

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* J. J. Thomson "Conduction in Gases," pp. 602, 624.

It thus seems probable that thermoluminescence may be closely connected with the electrical activity of salts and oxides upon metals ; experiments are now in progress in connection with these phenomena, which it is thought will reveal their origin.

ABSTRACT.

A method of demonstrating to an audience both ionisation by collision and the reduction of the sparking potential by the presence of initial ionisation is described in the Paper.

A spherical bulb, in which an electrodeless discharge is excited, contains two coils of platinum wire, coated with lime and aluminium phosphate respectively, which can be heated by means of a current. Within the range of pressure for which the electrodeless discharge can be excited, provided the discharge is not too intense, both the lime and aluminium phosphate, on being raised to incandescence, give a considerably increased effect, showing that the ions given off by these substances are effective under the action of the induced E.M.F. in producing an enormous number of others by collision.

The lower limit of the range of pressure over which the discharge can be excited with a given induced E.M.F. is considerably extended by heating either the lime or the aluminium phosphate, the former, however, being much more efficient than the latter in this respect. This extension of range is an illustration of the fact that the sparking potential is diminished by the presence of ions, and by negative ions more than by positive.

If the lime and aluminium phosphate are subjected while cold to an

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* Willows and Picton, loc. cit.

intense discharge for some time they exhibit the phenomenon of thermoluminescence, the lime on being warmed gently giving out a golden yellow luminosity and the aluminium phosphate a blue. The effect with lime apparently increases with an increase of electrical activity caused by continued heating, which suggests that the thermo-luminescence and electrical activity are closely associated. This suggestion receives support from the fact that lime alone does not exhibit thermo-luminescence, and that both lime on platinum and lime on nickel, which are electrically active, do. Further experments are being made on this point, which the author thinks will throw light on the origin of the anomalous electrical activity of lime on platinum.

A demonstration of these experiments was given.

VI. *An Experimental Method for the Production of Vibrations on Strings illustrating the Properties of Loaded or Unloaded Telephone Cables. By* J. A. FLEMING, *M.A., D.Sc., F.R.S.*

RECEIVED NOVEMBER 28TH, 1913.

THE problem of determining the possible vibrations of a loaded string—that is, a flexible string having small masses attached to it at equal distances—is one to which many mathematicians have given attention. It suffices to mention the well-known discussion of the problem by Lagrange,* the full treatment of it by Lord Rayleigh,† and an interesting Paper by Mr. C. Godfrey‡ published in 1898.

Of late years the matter has acquired an electrical interest from the close analogy existing between the transmission of mechanical vibrations along a loaded string and the propagation of alternating electric currents along a telephone cable having inductance coils inserted in it at equidistant intervals. In seeking for experimental methods of illustrating the properties of such loaded cables it was natural to turn for assistance to the visible oscillations produced on strings loaded or unloaded.

Almost the only method hitherto used for creating sustained vibrations on strings has been the elegant device of F. E. Melde, in which a string attached to the prong of a tuning fork is set into stationary sympathetic vibrations when the length and tension of the string is adjusted so that the vibrations travel along it in a time which has some integer ratio to the periodic time of the fork.

The objections to this method are (1) that we cannot put more than a certain tension on the string without stopping the fork even if the latter is electrically driven, (2) that when long loaded strings are employed we need very large forks to provide a sufficiently low frequency in the impulses, (3) that we cannot alter the frequency of vibration except by changing the fork, (4) that the vibrations of the string take place only in one plane and are not very visible unless viewed from a certain standpoint.

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* "Mécanique Analytique," Vol. I., part 2, sec. VI., § III.

† "Theory of Sound," Vol. I., Chap. VI., 2nd edition.

‡ "On Discontinuities Connected with the Propagation of Wave Motion along a Periodically Loaded String," "Phil. Mag.," April, 1898, p. 356.

In thinking over these difficulties it occurred to me that the fork could with great advantage be replaced by a small continuous-current electric motor, the speed of which can be easily controlled and measured and made low or high enough to study the vibrations on a string of any length, loaded or not loaded, and with any reasonable applied tension.

The best arrangements were found to be as follows :—

On the shaft of a small continuous-current motor, say, $\frac{1}{8}$ H.P. or $\frac{1}{4}$ H.P., is attached a pulley or disc which has a crank pin (K) inserted on its outer face at a distance of about 0·5 in. from the centre (*see* Fig. 1). To this pin is attached a light crank shaft (C) which is connected at the other end to a rocking lever (L). A small hook (H) is attached to the crank shaft, quite close to the crank pin, and another at the far end, close to the junction with the top end of the rocking lever.

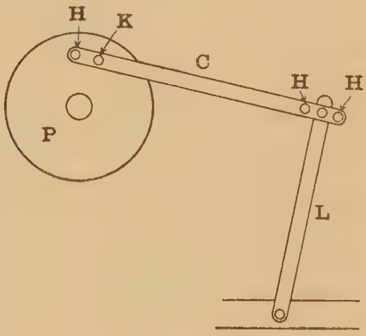

FIG. 1.

On the other end of the motor shaft is an endless screw gearing with a toothed wheel, the latter carrying a pin on its face which strikes once in every revolution against a small piece of steel clock-spring. This mechanism serves to count the speed of the motor, as we can take the time of 20 or 1,000 revolutions of the wheel by a stopwatch and, knowing the gear ratio, obtain the speed of the motor. The hook on the crank shaft near the crank pin describes, therefore, a circular path in space without rotation and the hook at the far end of the crank, shaft moves to and fro with an approximately simple harmonic motion.

If, then, we fasten a long stretched flexible cord to either hook we have the means of impressing upon the end of the cord simple harmonic motions either (1) in the direction of its length, or (2) transverse to its length, or (3) a circular motion equivalent to the combination of two simple harmonic motions differing 90 deg. in phase, each taking place in a direction at right angles to the direction of the stretched cord.

The remote end of the cord is attached to a pin which is screwed into a nut travelling on a long screw similar to the slide rest of a lathe (*see* Fig. 2). We can by this means put any desired tension on the cord, and also measure the tension by a spring balance inserted between the end of the cord and the terminal pin. If, then, we attach one end of the cord to the hook nearest the crank pin of the motor with the string stretched parallel to the axis of the motor and apply a certain tension at the other end of the string, and set the motor in rotation, we shall have vibrations propagated along the cord such that each part of the cord describes a circular motion. By properly adjusting the tension and the speed of the motor stationary oscillations may be set up on the cord which are the resultant of two simple transverse harmonic motions at right angles and differing 90 deg. in phase. The cord, therefore, when vibrating appears spindle shaped in virtue of the persistence of vision, and if the tension is adjusted we can make the cord vibrate in a single loop or in 1, 2, 3, &c., loops up to a dozen or more segments corresponding to the various harmonics of the fundamental mode of vibration of the cord (*see* Fig. 2).

By the slide-rest arrangement the tension can be accurately adjusted so as to produce any required number of sections and keep them steady. By sending the beam from an arc lamp lantern along the cord these stationary vibrations can be rendered visible to large audiences. We can easily measure the half wave-length or distance from node to node by a beam compass and having also the means of measuring the speed of the motor we can calculate the velocity with which a wave motion of any wave-length producible travels along the cord.

Arrangements can also easily be made to apply a tension to the cord by means of a weight attached to the end in the usual manner by passing the end of the cord over a fixed pulley.

The cord can be loaded by fixing on it at equal intervals glass beads or cork or wooden balls, and the possible modes of vibration on this loaded cord studied. By loading half of the cord or a middle portion, we can examine the reflection effects taking place when a vibration passes from one medium to another of different mean density. The following is the simple analytical treatment of the problem.

Let us assume that the mass of each load on the cord is m, in which we must include the mass of the length of cord intervening between two loads. Let d be the distance between the masses. Suppose also that in any shape into which the cord is thrown in its vibrations the angle made by any section of the cord with the straight line joining the extremities of the cord is so small that we may take the sine of this angle as equal to its tangent. Also let T be the tension of the cord, and let y_n be the transverse displacement of the nth mass. Then, following a method used by Prof. Tait,* we have for the equation of motion of that mass

$$m\frac{d^2y}{dt^2}=\mathrm{T}\left(\frac{y_{n+1}-y_n}{d}\right)-\mathrm{T}\left(\frac{y_n-y_{n-1}}{d}\right), \quad . \quad . \quad . \quad (1)$$

which expresses the fact that the difference of the components of the tensions taken perpendicular to the undisturbed cord on either side of any mass is the moving force on it. If we put $y_n=\mathrm{Y}\cos(pt-qx)$, where $p=2\pi/\tau$ and $q=2\pi/\lambda$, τ and λ being the periodic time and wave-length of the motion respectively, then on substitution in (1) we have

$$-mp^2\cos(pt-qx)=\frac{\mathrm{T}}{d}\{\cos(pt-q(x+d))-2\cos(pt-qx) + \cos(pt-q(x-d))\} \quad (2)$$

or

$$p^2=\frac{4\mathrm{T}}{md}\sin^2\tfrac{1}{2}qd. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Writing n for $p/2\pi$ and V for $n\lambda$ we have

$$\mathrm{V}=n\lambda=\sqrt{\frac{\mathrm{T}}{m/d}}\cdot\frac{\sin \pi d/\lambda}{\pi d/\lambda}. \quad . \quad . \quad . \quad . \quad (4)$$

The above is a correct analysis if the vibrations are in one plane, but as regards the case under consideration, viz., when each portion of the string is moving in a circle having its plane perpendicular to the line of the undisturbed cord, we must arrive at the equation of motion in another way.

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* *See* article "Wave," "Encyclopædia Britannica," 9th edition.

Since each load then describes a circle of constant radius y, the centrifugal force on a mass m moving with angular velocity p is mp^2y; and if we consider the load at y it is $mp^2y \cos (pt - qx)$. This force, however, is balanced by the sum or difference of the resolved parts of the tensions on either side of it, taken perpendicular to the axis of revolution. Hence we have

$$mp^2y_n = \mathrm{T}\left(\frac{y_n - y_{n-1}}{d}\right) - \mathrm{T}\left(\frac{y_{n+1} - y_n}{d}\right)$$

and by substitution for y_n of $\mathrm{Y} \cos (pt - qx)$ gives us the same equation (2) and hence (4) follows, as above.

If d/λ is very small, that is, if the loads are very close together, then $m/d = \rho$ is the average density per unit of length, and the formula reduces to $\mathrm{V}_0 = \sqrt{\mathrm{T}/\rho}$, which is the known expression for the velocity of propagation of a wave along a cord of mass per unit length ρ and longitudinal tension T. Hence, if we call this latter the wave velocity corresponding to the uniformly distributed or smoothed out masses we have finally

$$\mathrm{V} = \mathrm{V}_0 \frac{\sin \pi d/\lambda}{\pi d/\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

Also (5) can be written

$$n = \frac{1}{\pi d}\sqrt{\frac{\mathrm{T}}{m/d}} \sin \pi d/\lambda \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

From which it is seen that no frequency greater than $\sqrt{\mathrm{T}}/\sqrt{\pi^2 md}$ can be propagated along the loaded cord. This corresponds to about three loads per wave, or when $d = \lambda/2$.

The values of $(\sin \pi d/\lambda)/(\pi d/\lambda)$ in terms of λ/d or the number of loads per wave are given in the Table I. below :—

TABLE I.

λ/d.	$(\mathrm{Sin}\ \pi d/\lambda)/(\pi d/\lambda)$.	λ/d.	$(\mathrm{Sin}\ \pi d/\lambda)/(\pi d/\lambda)$.
1	0	6	0·955
2	0·637	7	0·967
3	0·827	8	0·974
4	0·900	9	0·980
5	0·935	10	0·984

It is obvious that this ratio runs rapidly up towards unity and that even when λ/d is as small as 10 the discontinuous loading is equivalent in practical effect to continuous loading, as if the added mass was uniformly distributed along the string.

These deductions from theory can be easily illustrated and confirmed by the use of the vibrating string apparatus above described.

The best cord to use with it is a light inelastic cord called cotton cord, and a useful diameter is about 1 mm. or 2 mm. The sample I have used has a weight of 0·00839 gramme per centimetre.

A length of 2 or 4 metres was employed and one end attached to the motor, the other being led over a light aluminium pulley well pivoted and terminated in a scale pan in which weights were placed. Using appropriate tensions and frequencies the cord was thrown into stationary vibrations giving 1, 2, 3, 4, &c., half-wave lengths or loops. The frequency N (per second) and wave-length λ were measured and also the tension T of the cord in grammes weight, reduced to dynes by multiplication by 981. The product $N\lambda$ was then compared with the quotient $\sqrt{T}/\sqrt{\rho}$, where $\rho = 0{\cdot}00839$. The following Table II. gives the results :—

TABLE II.—*Transverse Vibrations of an Unloaded String.*
ρ = weight per cm. = 0·00839 gramme. Length = 2 metres.

Number of half waves or loops.	Tension of string = T.		Frequency of oscillations per sec. = N	Wave-length in cms. = λ.	Velocity of propagation.	
	Grammes.	Dynes.			$N\lambda$.	$\sqrt{T}/\sqrt{\rho}$.
1	1,000	981,000	27·9	390	10,900	10,850
2	335	329,000	31·8	200	6,360	6,280
3	150	147,000	32·9	130	4,280	4,200
4	87	85,400	32·9	98	3,220	3,220
5	50	49,100	30·7	79	2,320	2,420
With a cord 4 metres in length vibrations up to the 10th harmonic could be obtained giving from 10 to 19 loops or half-waves.						
10	55	54,000	31·2	80	2,496	2,540
19	15	14,710	32·2	40	1,288	1,320

It will be seen that the wave velocity given by product $N\lambda$ is approximately equal to the quotient $\sqrt{T}/\sqrt{\rho}$. There is a certain difficulty in measuring the wave-length as the nodes are not very sharply defined, but the agreement is fairly good and proves that the velocity of propagation of the wave varies as the square root of the tension and inversely as the square root of the density of the cord.

A second set of measurements was made employing a constant tension of 110 grammes on the cord and variable speeds of the motor to produce different wave-lengths, and the results are embodied in Table III. :—

TABLE III.—*Transverse Vibrations of an Unloaded String.*
ρ=weight per cm.=0·00839 gramme. Length=2 metres.

Number of half waves or loops.	Tension of string =T.		Frequency of oscillations per sec.=N	Wave-length in cms. =λ.	Velocity of propagation.	
	Grammes.	Dynes.			Nλ.	$\sqrt{T}/\sqrt{\rho}$.
1	110	108,000	8·5	390	3,310	3,580
2	110	108,000	17·8	201	3,580	3,580
3	110	108,000	26·1	134	3,500	3,580
4	110	108,000	36·1	98	3,540	3,580

Length of cord 4 metres.

Number of half waves or loops.	Grammes.	Dynes.	Frequency of oscillations per sec.=N	Wave-length in cms. =λ.	Nλ.	$\sqrt{T}/\sqrt{\rho}$.
2	110	108,000	8·7	404	3,520	3,580
4	110	108,000	17·8	201	3,580	3,580
6	110	108,000	26·9	134	3,600	3,580
8	110	108,000	36·7	98	3,600	3,580

These last results similarly prove that within the limits of wave-length employed the velocity of propagation is independent of the frequency and equal to the quotient $\sqrt{T}/\sqrt{\rho}$.

On the cord 4 metres in length there was no difficulty in producing as many as 20 half-waves quite stationary, each having a length of 20 cm.

The next step was to prepare a loaded cord by placing on the same cotton cord glass beads each weighing 0·208 gramme at distances of 20 cm. The mean density or weight per centimetre length of cord was thereby raised to 0·0188 gramme. The same experiments were then repeated with this loaded cord using variable tensions (Table IV.), and constant tension (Table V) with the results given.

TABLE IV.—*Transverse Vibrations of a Loaded String.*
Loads of 0·208 gramme at intervals of 20 cm. Mean weight per cm.=ρ=0·0188 gramme. Length of cord=4 metres. Variable tension.

Number of half waves or loops.	Tension of string =T.		Frequency of oscillations per sec.=N	Wave-length in cms. =λ.	Velocity of propagation.	
	Grammes.	Dynes.			Nλ.	$\sqrt{T}/\sqrt{\rho}$.
3	810	795,000	25·0	266	6,650	6,510
4	550	540,000	26·8	200	5,360	5,360
5	380	373,000	28·0	158	4,420	4,460
6	265	260,000	29·1	130	3,790	3,720
7	205	201,000	29·1	110	3,210	3,280
8	180	177,000	31·2	97	3,020	3,060

TABLE V.

Same loaded string as in Table IV., but with constant tension.

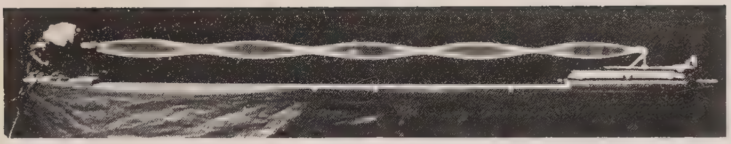
Number of half waves or loops.	Tension of string = T.		Frequency of oscillations per sec. = N	Wave-length in cms. = λ.	Velocity of propagation.	
	Grammes.	Dynes.			Nλ.	$\sqrt{T}/\sqrt{\rho}$.
2	210	206,000	8·1	400	3,240	3,310
3	210	206,000	12·8	266	3,400	3,310
4	210	206,000	16·6	200	3,320	3,310
5	210	206,000	19·9	160	3,200	3,310
6	210	206,000	24·6	132	3,240	3,310
7	210	206,000	28·6	110	3,160	3,310

It will be seen that here also the wave velocity measured by the product $N\lambda$ is approximately equal to the quotient $\sqrt{T}/\sqrt{\rho}$ and that the wave velocity is independent of the frequency.

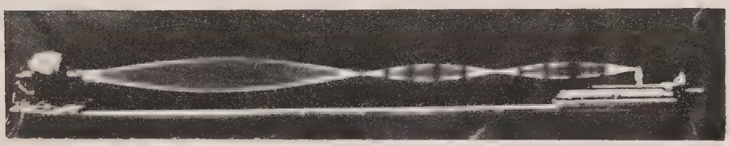
The loaded cord, however, differs from the unloaded in this respect, that whereas there was no difficulty in producing on the unloaded string as many as 19 or 20 stationary half-waves or loops, each about 20 cm. in length, it was found quite impossible to produce waves as short as this on the loaded string with loads 20 cm. apart. The latter refuses to oscillate sufficiently quickly and will not propagate waves of a higher frequency than corresponds to a wave-length of rather more than double the distance between the loads. On putting on a higher frequency at one end of the cord the vibrations or loops on it are seen to die away quickly, but sustained stationary oscillations on the whole length of the loaded string cannot be set up unless the frequency is less than the critical value.

This method of exciting the oscillations on a string can also be employed to exhibit in a beautiful manner the wave reflection that takes place at the junction between two strings of equal tension but unequal mass per unit of length or at the junction between an unloaded and a loaded string.

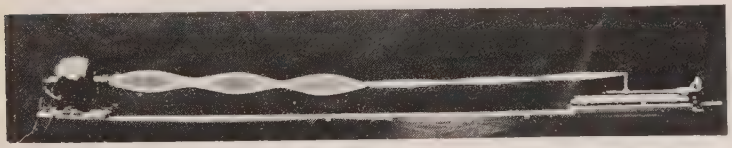
If we place glass beads at regular intervals on one-half of a string 4 metres long and attach the extremity of the unloaded half to the motor wheel and apply a tension at the remote end, then standing waves will be produced on both parts when the tension is adjusted (*see* II., Fig. 2). These standing waves have, however, different wave-lengths on the two parts which are inversely as the square roots of the mean cord densities. Thus in one experiment tried the standing waves had a wave-length of 100 cm. on the unloaded half of the string and a wave-length of 66 cm. on the loaded half. These two parts of the



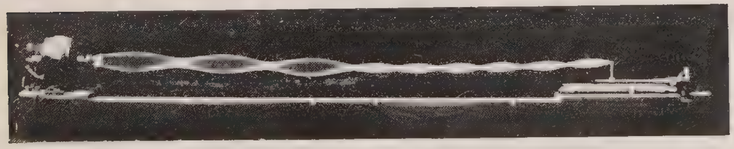
I.—Standing Waves on Unloaded String.



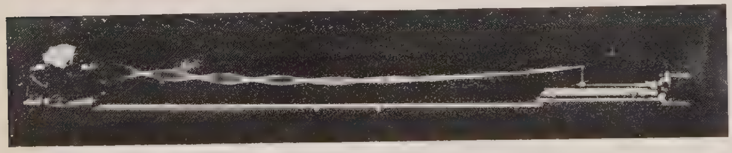
II.—Standing Waves on String Half Loaded, Half Unloaded.



III.—Reflection of Wave by Load at Centre of String.



IV.—Standing Waves on String Half Single, Half Fourfold.



V.—Standing Waves on Loaded String, Wave-length Covers about Three Loads.

Fig. 2.

To face p. 68.

string had mean densities of 0·00839 and 0·0188 gramme per centimetre respectively. The ratio of the square roots of these last numbers is as 29 is to 43·3 and 29×100=2,900 whilst 43·3×66=2,858. Hence the wave-lengths are nearly inversely as the square roots of the mean cord densities. The amplitude of the waves on the unloaded half of the cord was, however, much greater than that of the waves on the loaded cord, thus showing the loss of amplitude on reflection. The same thing can be better shown by employing a cord one half of which is single and the other half made of four similar cords in parallel and tied together at intervals. When this is vibrating we have waves on the four-fold part which are half the wave length of those on the single part (*see* IV., Fig. 2).

Again, if the tension was decreased slightly so as to lower the velocity and decrease the length of the waves a point was soon reached at which whilst standing waves were produced on the unloaded half of the cord, no waves were produced on the loaded half. This happened when the length of the waves on the unloaded half was about 60 cm. and therefore those on the unloaded half should have been about 40 cm., or equal in length to twice the distance between the loads ; so that one wave would cover three loads or a little more.

This experiment proves clearly that there is an inferior limit to the possible wave-length and a superior limit to the possible frequency of the vibrations which can be produced on the loaded string exactly as predicted by theory. The same reflection of the waves and loss of amplitude on passing a junction between two uniform cords of different mass per unit of length can be shown by joining two cords of equal length, one a plain cotton cord and the other a piece of whipcord or heavy gilt thread, and attaching one end of the length to the crank of the motor and applying a tension to the other end. By adjusting this tension it is easy to show the loss in amplitude of the wave in passing such a junction, or even the total reflection of the wave (*see* III., Fig. 2). This phenomenon has its electrical analogue in the loss in amplitude or reflection of the electrical waves at the junction between two conductors, say, an unloaded land line and a submarine cable, or loaded line, the two having different capacities and inductances per mile when telephonic currents are transmitted through such a circuit.

Again, we can show by this vibration apparatus the advantage of tapering off the loading in the case of a junction between a loaded and an unloaded line owing to the diminution of the reflection losses. When a wave passes from one medium to another of different refractive index, and if this index changes somewhat suddenly at the junction plane we know that wave reflection takes place. If, however, the index changes gradually the reflection losses are much reduced or else annulled. Hence, if a loaded telephone cable is connected to an unloaded cable partial reflection of the wave takes place at the junction. If, however, the loading is tapered off, these reflection losses are much reduced. This can be visibly illustrated by the vibrating string apparatus very easily. If we load one-half of a string with glass beads but taper off the loading by using towards the middle point beads or knots of gradually decreasing mass, and then attach one end of the unloaded half to the motor crank and apply a tension to the other end of the whole cord we shall see that the reflection losses at the junction are much reduced.

The partial reflection and consequent loss of amplitude in the transmitted wave can even be shown by placing a single bead or load of appropriate mass in the centre of an otherwise unloaded string. If one end of the string is vibrated and a suitable tension applied we can produce stationary waves of large amplitude on that part of the string between the motor and the load, but the amplitude of the vibrations produced on the far side of the load is much less, thus showing the partial loss of amplitude in the transmitted vibration owing to the reflection losses.

In conclusion, a short method may be given for arriving at an expression for the velocity of propagation of an electric wave along a coil-loaded telephone cable, which was first given by Prof. I. Pupin in his important memoirs on this subject,* but which may be obtained more simply in the following manner :—

Let us assume we have a two-wire cable having capacity C and leakance S due to the dielectric between the wires, both reckoned per unit length of the circuit. Let us suppose that loading coils, each having inductance L_1 and resistance R_1, are inserted at intervals d along the cable. Also let the cable itself have a conductor resistance r and inductance l per unit of length. Let us denote the currents through three successive loading coils by i_{n-1}, i_n and i_{n+1}, and the P.D. between the wires by v_1 at a point half-way between the $(n-1)$th and nth coil, and by v_2 half-way between the nth and $(n+1)$th coil. Furthermore, let the currents and potentials be simple harmonic functions of the time and vary as the real part of ε^{jpt}, where $j=\sqrt{-1}$ and $p=2\pi$ times the frequency. Hence $d/dt=jp$ and $d^2/dt^2=-p^2$.

---

* *See* I. Pupin, "Transactions of the American Institute of Electrical Engineers," Vol. XVI., p. 93, 1899, and Vol. XVII., p. 445, 1900. Also *see* J. A. Fleming, "The Propagation of Electric Currents in Telephone and Telegraph Conductors," 2nd edition, p. 110.

Consider, then, the section of the cable between the $(n-1)$th and nth loading coil. The average gradient of the current in that section of the cable is $(i_{n-1}-i_n)/d$, and if this gradient line is nearly straight the gradients must be equal to $(S+jpC)v_1$.

Also in the same manner for the next section we shall have $(i_n-i_{n+1})/d$ equal to $(S+jpC)v_2$.

Therefore, taking the difference we have

$$\frac{(i_{n-1}-i_n)-(i_n-i_{n+1})}{d}=(S+jpC)(v_1-v_2). \quad . \quad . \quad (7)$$

But if the impedance of each coil is large compared with that of the line between the coils, we can say that,

$$v_1-v_2=(R_1+jpL_1)i_n \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

If, however, we include the resistance r and inductance l of the cable itself apart from those of the loading coil we must write this last equation in the form

$$v_1-v_2=i_n\left\{\left(r+\frac{R_1}{d}\right)+jp\left(l+\frac{L_1}{d}\right)\right\}d. \quad . \quad . \quad . \quad (9)$$

In actual practice the inductance of the length d of the cable between the loading coils is negligible compared with that of the loading coil, and the resistance of the loading coil is small compared with that of the length of cable between the loading coils. Hence, it is sufficient to write the above equation in the form

$$v_1-v_2=i_n(R+jpL)d, \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where R and L are the average resistance and inductance of the loaded cable per unit of length of the circuit. Hence, from (7) and (10) we have

$$i_{n-1}-2i_n+i_{n+1}=i_n(S+jpC)(R+jpL)d^2. \quad . \quad . \quad (11)$$

If the current is a simple periodic current then i_n may be taken as proportional to $\cos(pt-qx)$, where $p=2\pi/\tau$, and

$q=2\pi/\lambda$ and $x=nd$. The equation (11) then becomes, by substitution,

$$\cos(pt-q(x-d))-2\cos(pt-qx)+\cos(pt-q(x+d)) \\ =\cos(pt-qx)\{\mathrm{RS}-p^2\mathrm{LC}+jp(\mathrm{CR}+\mathrm{LS})\}d^2 \quad . . \quad (12)$$

But since $-p^2=(jp)^2$ equation (12) can be written

$$(jp)^2+2\left(\frac{\mathrm{R}}{2\mathrm{L}}+\frac{\mathrm{S}}{2\mathrm{C}}\right)jp=-\left(\frac{4}{\mathrm{LC}d^2}\sin^2\tfrac{1}{2}qd+\frac{\mathrm{RS}}{\mathrm{LC}}\right) \quad . . \quad (13)$$

Solving this quadratic we have

$$jp=-\left(\frac{\mathrm{R}}{2\mathrm{L}}+\frac{\mathrm{S}}{2\mathrm{C}}\right)\pm j\sqrt{\frac{4}{\mathrm{LC}d^2}\sin^2\pi d/\lambda-\left(\frac{\mathrm{R}}{2\mathrm{L}}-\frac{\mathrm{S}}{2\mathrm{C}}\right)^2}. \quad (14)$$

Hence the frequency of the oscillations is given by

$$p/2\pi=f=\frac{1}{\pi d}\sqrt{\frac{1}{\mathrm{LC}}\sin^2\pi d/\lambda-\left(\frac{\mathrm{R}d}{4\mathrm{L}}-\frac{\mathrm{S}d}{4\mathrm{C}}\right)^2}. \quad . \quad (15)$$

Hence the velocity $\mathrm{W}=f\lambda$ with which the wave travels is given by

$$\mathrm{W}=\sqrt{\frac{1}{\mathrm{LC}}\,\frac{\sin^2\pi d/\lambda}{(\pi d/\lambda)^2}-\frac{\lambda^2}{4\pi^2}\left(\frac{\mathrm{R}}{2\mathrm{L}}-\frac{\mathrm{S}}{2\mathrm{C}}\right)^2}. \quad . \quad . \quad . \quad (16)$$

If, then, the primary constants R, L, S and C are so related that $\mathrm{R}/\mathrm{L}=\mathrm{S}/\mathrm{C}$, that is, if the cable is sufficiently loaded to be distortionless, we have

$$\mathrm{W}=\frac{1}{\sqrt{\mathrm{LC}}}\,\frac{\sin\pi d/\lambda}{\pi d/\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Accordingly the cable will not transmit a wave if the frequency f is increased to a point at which f is greater than $1/\pi d\sqrt{\mathrm{LC}}$, that is, if the periodic time T is less than $\pi\sqrt{\mathrm{L}d\,.\,\mathrm{C}d}$. If the wave-length is made less than the coil or load interval d, then, as shown by Mr. Godfrey (*loc. cit.*), there are curious discontinuities as the wave-length is progressively decreased. We are presented with a phenomenon analogous to the formation of absorption lines in spectra, in which a medium is transparent or more or less opaque in accordance with the wave-length of the incident light. If, in the formula (15) above, we put $\mathrm{S}=0$, it becomes identical with the formula given by Pupin. If we put both R and S equal to zero, and if d/λ is small, then the wave velocity becomes $1/\sqrt{\mathrm{LC}}$, and is the same as if the added inductance is uniformly distributed along the cable.

ABSTRACT.

Prof. Fleming exhibited an apparatus for the production of vibrations on strings loaded or unloaded. This method is an improvement on that of Melde, in which transverse vibrations are created on a string attached to the prong of a tuning fork.

Melde's method has the disadvantage that it is not possible to put great tension on the string without stopping the fork, or to use very heavy or loaded strings without using forks of great size. Moreover, the frequency cannot be varied except by changing the fork.

In the Author's method the vibrations are produced on a string by attaching one end to the shaft of a small continuous-current motor of about $\frac{1}{8}$ H.P. The motor has on one end of its shaft a mechanism for counting revolutions, and on the other end a disc which has a crank pin inserted in its outer face, and to this pin is attached a light crankshaft connected at its outer extremity with a rocking lever. The string is fastened to a hook on the crankshaft near to the crank pin.

The other end of the string is attached to a fixed point which can be moved by means of a screw, in some cases a spring balance being interposed to measure the tension.

When the motor is started the string has a circular motion given to its end which is equivalent to two simple harmonic motions at right angles to each other.

If the tension is rightly adjusted the string then vibrates in sections, and the number of sections can be adjusted by altering the tension. The distance from node to node can then easily be measured and the frequency determined from the speed of the motor. In this way the velocity of the wave is measured, and can be compared with the velocity determined by taking the square root of the quotient of the tension by the linear density of the string.

A number of tables were exhibited showing the velocity of the wave motion on strings of various kinds confirming the known laws of string vibrations.

This method is particularly useful in studying the properties of loaded strings. Strings made of flexible cotton cord can be loaded with glass or wooden beads and set in vibration. In this manner it can be shown experimentally that when the wave length on the string extends over a distance of more than 8 or 10 loads, the string vibrates as if the loading matter were uniformly distributed, but the string cannot propagate vibrations when the half wave length approaches equality to the distance between two loads.

It is also possible to show by this apparatus very prettily the reflection of a wave at a load placed at any point on the string, and also that this reflection is reduced by tapering off the loading.

We can thus imitate with this loaded vibrating string all the phenomena of inductive loading in telephone cables on the Pupin system.

The theory of the vibrations of a loaded string has been studied by many mathematicians—for example, by Lagrange, Lord Rayleigh, Prof. P. G. Tait, and a very interesting Paper was published on the subject by Mr. Charles Godfrey in the " Philosophical Magazine " for April, 1898.

This method of causing a string to vibrate by a motor affords a simple method of demonstrating all the laws of vibrating strings to a class

and also can be employed to show experimentally results of loading telephone cables in various ways, and the reflections that occur when cables with different constants are connected in series.

DISCUSSION.

Dr. Russell complimented Prof. Fleming on his short and easy proof of Pupin's law.

Mr. Campbell remarked that Prof. Tait, of Edinburgh, used to have a hand machine with which he could produce eight or nine loops on a long vibrating string. But the success of his demonstrations depended largely upon the skill of the operator.

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